

OPTIMAL SCHEDULING
OF
HYDRO AND HYDROTHERMAL
POWER SYSTEMS

*

by *aughan*
GARRY V. JONES

B.E. (Hons.), M.I.E. Aust.

University of Tasmania

*

An investigation carried out within the
Department of Electrical Engineering
University of Tasmania

*

Submitted for the Degree of
Doctor of Philosophy

*

FEBRUARY 1982
(March 1983)

SUMMARY

In a power system with only thermal generation, fuel stocks are usually adequate for any generation schedule permitted by plant ratings. This is not so for hydro generation, whose energy availability is determined by water storages and streamflows and ultimately by the weather. In scheduling a system with hydro-generation, this extra limitation must be recognised, and it dictates the form of the scheduling method if the proportion of hydrogeneration is high. The optimal schedule is taken as the one supplying the power demand at minimum total cost or resource use over a specified period with adequate reliability, security and quality of supply, subject to physical, operating and statutory constraints.

Optimisation of a hydro or hydrothermal system comprises a family of interacting problems characterised by differing time scales, degrees of detail and accuracy of information. The shorter the time span, the greater the detail and data accuracy. The family includes future planning over a decade or so, annual optimisation over a water year, water management over a period of days or weeks, daily optimisation over a load cycle and instantaneous regulation. This work is concerned with the study of the short term (daily) scheduling of generating plant in power systems with only hydro generation or of integrated (hydro-thermal) power systems with a significant proportion of hydro generation.

Short-term optimisation of generation for a hydro or hydrothermal power system is a very large variational problem with

many operational and physical constraints. The normal approach to its solution has been to apply optimal control or mathematical programming techniques to transform the problem into one of solving a set of equations characterising the optimum. Even for systems of moderate size, solution times for these equations are often high and intermediate stages of the solution may not provide feasible schedules. This thesis presents an alternative strategy by which successive feasible schedules approaching optimality are obtained rapidly.

The hydrothermal scheduling problem is variational because of the need to consider overall rather than just instantaneous water use. In the optimisation the power system is modelled by differential equation equality constraints describing the river dynamics, and algebraic constraints describing the transmission system and thermal plants. Although real and reactive system loads and reservoir inflows are stochastic, in the short-term, they may be treated as deterministic without substantial penalty.

The scheduling process presented in this thesis starts by committing hydro units and finding a starting schedule by a global search for the step-loading schedule which minimises generation losses and goes some way towards minimising transmission losses. The discussions of the search are reduced by reordering the system load curve and permitting only such combinations of steps in generator loading as are found to characterise optimal step-loading schedules on the reordered load curve. This starting schedule obeys the water use constraints and will, in many cases constitute a feasible schedule.

After unit commitment of hydro plants has been fixed by the step-loading schedule, the solution of the integrated problem is approached through a sequence of smaller problems in which alternately only hydro and only thermal plants are rescheduled with reference to the same goal. The initial step-loading schedule is refined, employing optimal load flow computations to schedule thermal generation, VAR allocation and regulator settings, alternating with a gradient or Newton search which uses the optimal load flow results to improve the hydro schedule.

The optimal load flow algorithm which schedules thermal plants and ensures conformity with electrical system constraints yields dual variables which are the sensitivities of the objective function to the cost of power flows across hydro plant busbars. The incremental costs of supplying the demand may therefore be calculated in terms of hydro discharge rates. The hydro plants are then rescheduled by a hill-climbing step with hourly flows as independent variables, constrained by the specified reservoir depletions. Each step establishes new power flows at the hydro busbars, which are held during the next optimal load flow computation. In this way, feasible, properly constrained schedules are obtained at the end of each set of optimal load flow solutions. Each step also provides a comprehensible measure of progress towards the optimum in the form of changes in cost and generation schedules. In preceding this local minimum finding search by a restricted global search for unit commitment, convergence to the globally optimal schedule is enhanced.

By arranging the optimisation around one of the existing

highly developed optimal load flow algorithms, much of the sensitivity information needed to tell an operator not only the optimal schedule, but also the effect of small changes on it, is available and if a Newton type search is used, the sensitivity of the objective function to total discharge of each hydro plant is also available with no further computation.

The method is, by virtue of its modular approach, able to make use of any future developments in the optimal load flow field which is still being intensively researched, with additional or superior features being easily incorporated.

ACKNOWLEDGEMENTS

The work of this thesis was carried out in conjunction with the Department of Electrical Engineering, University of Tasmania. Financial support has been received from the Electrical Research Board of Australia and the Commonwealth Postgraduate Research Awards Scheme.

I wish to thank my supervisor, Dr. John Norton, for his assistance and encouragement during this project, together with other members of the department who have been of assistance.

The load control section of the Hydro-Electric Commission of Tasmania has been most helpful, and the assistance of Mr. G. N. Kerrison, Mr. R. L. Adams and their staff is gratefully acknowledged.

This work has been made possible by my employers, Associated Pulp and Paper Mills Limited, who kindly allowed me extended leave for the initial four years of this project.

*

To the best of my knowledge, this thesis contains no copy of material previously published or written by any other person, or material which has previously been submitted for the award of any other degree or diploma, except where due reference is made in the text.



G. V. JONES

CONTENTS

	Page
<u>Summary</u>	ii
<u>Acknowledgements & Declaration</u>	vi
<u>Contents</u>	vii
 <u>Chapter One: INTRODUCTION AND THESIS PLAN</u>	 1
1.1 Hierachies in power system planning, operation and control	3
1.2 Scope and plan of thesis	6
1.3 References	12
 <u>Chapter Two: A HISTORICAL DEVELOPMENT OF OPTIMAL CONTROL IN HYDROTHERMAL POWER SYSTEMS</u>	 15
2.1 Conventional approaches to thermal scheduling	18
2.1.1 Merit order method	19
2.1.2 Equal incremental costs method	19
2.1.3 Equations of co-ordination methods	20
2.2 Conventional approaches to hydrothermal scheduling	22
2.2.1 Maximum efficiency method	23
2.2.2 Equal incremental costs method	23
2.2.3 Equations of co-ordination method	24
2.2.4 A practical comparison of conventional methods of hydrothermal-system scheduling	26
2.3 Quasi-conventional approaches to hydrothermal scheduling	26
2.3.1 Linear programming	28
2.3.2 Dynamic programming	28
2.3.3 Pontryagin's maximum principle	29
2.4 Weaknesses of conventional and quasi-conventional optimisation methods in hydrothermal-system scheduling	31

	Page
2.5 Hydrothermal scheduling based on an accurate system model	32
2.5.1 Nonlinear programming with decomposition	32
2.5.2 Initial value method based on a generalised maximum principle	33
2.5.3 Conjugate gradients method based on a generalised maximum principle	35
2.6 Summary	36
2.7 References	37
 <u>Chapter Three: A MATHEMATICAL MODEL OF THE HYDROTHERMAL POWER SYSTEM FOR SHORT-TERM OPTIMISATION</u>	 46
3.1 The optimality criterion	46
3.2 Selection of control variables	48
3.3 Constraints on the system	49
3.3.1 The load-balance constraints	49
3.3.2 Inequality constraints on the electrical subsystem	53
3.3.3 Flow balance constraints on the hydro-reservoirs	55
3.3.4 Inequality constraints acting on the hydro subsystem	56
3.3.5 Hydro plant power vs discharge rate characteristics	57
3.3.6 Physical and imposed constraints	58
3.4 Representation of hydro-electric and thermal power plants	59
3.4.1 Representation of hydro-electric power plants	61
3.4.1.1 Load division between units on line	61
3.4.1.2 Variation of the number of turbines in service	69
3.4.1.3 Mathematical description of hydro-electric power plants	70

	Page
3.4.2 Representation of thermal power plants	71
3.4.2.1 Load division between boilers and turbines on line	72
3.4.2.2 Variation of the number of boilers and turbines in service	75
3.4.2.3 Mathematical description of a thermal power plant	77
3.5 Recapitulation	78
3.6 References	
 <u>Chapter Four: A STEP-LOADING TECHNIQUE FOR UNIT COMMITMENT IN THE HYDRO SUBSYSTEM</u>	 84
4.1 Step-loading in an ideal hydro subsystem	85
4.1.1 Maximum efficiency operation of a single unit hydro plant	86
4.1.2 Maximum efficiency operation of a multi- unit hydro plant	86
4.2 Application of step-loading to non-ideal systems with variable transmission loss	87
4.3 Computational aspects of step-loading	91
4.3.1 Five plant hydrothermal and hydro system examples	93
4.3.2 The multi-minima nature of step-loading	101
4.4 Case study	107
4.4.1 Experimental results	111
4.5 Effects of head variations	115
4.6 Discussion	116
4.7 References	119
 <u>Chapter Five: REFINEMENT OF THE STEP-LOADING SCHEDULE BY RELAXATION USING A DECOMPOSITION TECHNIQUE</u>	 121
5.1 Decomposition of the hydrothermal power system into subsystems	123

	Page
5.2 The proposed method of schedule refinement	124
5.3 The choice of a hill-climber for co-ordination of the electrothermal and hydro subsystems	126
5.3.1 Gradient hill-climbers	128
5.3.2 Second-derivative hill-climbers	131
5.4 Monte-Carlo tests of hill-climber performance	132
5.5 Summary	136
5.6 References	

Chapter Six: COMPUTATIONAL METHODS FOR SCHEDULING THE ELECTROTHERMAL SUBSYSTEM

	140
6.1 The method of Dauphin, Feingold and Spohn	142
6.2 The method of Nabona and Freris	145
6.3 The method of Carpentier and Sirieux	147
6.4 The method of Dommel and Tinney and related methods	152
6.5 The sequential unconstrained minimisation technique	158
6.6 The sequential linearly constrained minimisation technique	159
6.7 Methods scheduling reactive generation only	161
6.8 Recapitulation	163
6.9 References	164

Chapter Seven: OPTIMAL SCHEDULING OF THE ELECTROTHERMAL SUBSYSTEM

	168
7.1 The load-flow equations	170
7.2 Feasible solutions to the load-flow	171
7.2.1 Implementation of the Newton load-flow	171
7.3 Optimal load-flow without inequality constraints	179
7.4 Inequality constraints on control variables	180
7.5 Functional inequality constraints	181
7.5.1 External penalty function method	182
7.5.2 The sequential unconstrained minimisation technique	184
7.6 Recapitulation	185
7.7 References	188

	Page
<u>Chapter Eight: THE INTEGRATED SYSTEM</u>	191
8.1 Representation of the electrothermal subsystem	192
8.2 Integration of the hydro and electrothermal subsystems	201
8.2.1 A simplification to increase computing efficiency	204
8.3 Example problem illustrating scheduling of the integrated system	207
8.4 Discussion	221
8.5 References	222
 <u>Chapter Nine: SENSITIVITY CONSIDERATIONS IN A HYDRO-THERMAL POWER SYSTEM</u>	 223
9.1 The electrothermal subsystem	225
9.1.1 Sensitivity relations between electrical variables	225
9.1.2 Sensitivity of the objective function to the electrical variables	228
9.1.3 Rescheduling the electrothermal subsystem	232
9.1.4 Performance index deterioration resulting from inaccurate knowledge of parameters	234
9.1.5 Example problem illustrating electrothermal subsystem sensitivity calculations	239
9.2 The hydro subsystem	241
9.2.1 The interface between hydro and electrothermal subsystem schedules	242
9.2.2 Sensitivity of the objective function to hydro variables	243
9.2.2.1 Interpretation of the dual variables	245
9.2.2.2 Rescheduling of the hydro subsystem	247
9.2.3 Example problem illustrating hydro subsystem sensitivity relations	249
9.3 Sensitivity to the size of the scheduling intervals	257

	Page
9.4 Summary	259
9.5 References	261
 <u>Chapter Ten: CONCLUSIONS AND FURTHER WORK</u>	 263
10.1 Summary	263
10.2 Conclusions	264
10.3 Proposals for further work	265
10.3.1 Extension of the step loading method to medium term scheduling	266
10.3.2 The stochastic nature of river flows and system loads	267
10.3.3 The state of the system	268
10.3.4 General	268
10.4 References	269
 <u>Appendix One: DERIVATION OF THE STEP-LOADING TECHNIQUE BY PONTRYAGIN'S MAXIMUM PRINCIPLE</u>	 270
A1.1 Maximisation of the overall efficiency of a fixed head hydro plant	270
A1.2 Optimisation of the daily load schedule in multi-plant systems	275
A1.2.1 Ideal systems	277
A1.3 References	279
 <u>Appendix Two: RESULTS OF MONTE-CARLO TESTS OF HILL- CLIMB ALGORITHM PERFORMANCE</u>	 280
 <u>Appendix Three: PAPERS FOR PUBLICATION</u>	 288
A3.1 Hydrothermal optimisation by refinement of step loading schedules	289
A3.2 Feasible short-term scheduling for hydro and hydrothermal power systems	294

1. INTRODUCTION AND THESIS PLAN

Increasing size and complexity of hydro and integrated or hydrothermal power systems have created a need for computer based methods in system optimisation at all levels, while improvements in computing and control hardware and techniques have brought optimal control and operation of systems much closer to practical realisation and are contributing substantially to improved system planning.

By optimal operation of a power system, we mean supplying all power demands at minimum cost or resource use and with adequate reliability, security and quality of supply, subject to physical, operational and statutory limitations. These aims are often in conflict, very high reliability involving increased cost. Ultimately, subjective judgements have a major role at both planning and operating levels. These include installation of sufficient generation and transmission capacity, appropriate scheduling of maintenance, provision of adequate spinning reserve, analysis of transmission contingencies and preservation of adequate operating stability margins.

In a purely thermal power system, there are usually sufficient fuel stocks to meet any generation schedule within the capabilities of the equipment. This is not the case in a predominantly hydro system, in which the energy availability and distribution is related to the medium and long-term weather patterns. The hydrothermal or hydro economic control must therefore be considered as a variational or optimal control problem in contrast to the static

optimisation involved in the purely thermal problem. We therefore have a cost functional rather than a cost function, and this is subject to differential equality constraints describing the river system dynamics as well as algebraic constraints describing the power system.

Hydro power plants fall into three distinct categories; run of river, peaking and storage. The run of river plants, having no storage facilities, must utilise flow as it arrives either from a natural river or from an upstream storage. Peaking plants have short-term pondage only, and storage plants have large seasonal storage reservoirs, storing water during high inflow periods for use when required. The pumped storage plant is a form of peaking plant, absorbing energy from the grid during off-peak hours rather than from a river system, and releasing it on peak loads.

Since hydro plants have relatively low maintenance and operating costs, a worthwhile saving of thermal fuel or storage plant water may be obtained by optimally scheduling the hydrothermal system. As noted in chapter 2, optimal operation and control of hydrothermal power systems has attracted interest since the 1920's, with digital computer based methods being introduced in the 1950's [5, 6]. A great impetus was given to work in both thermal and hydrothermal system scheduling by Capentier and Sirieux [1, 2] who developed the first complete formulation of the problem of optimal control in thermal systems and by Tinney and Hart [11] who developed the Newton's method load flow concepts of Van Ness [13] into a fast practical technique generating sensitivity coefficients useful in

economic scheduling. The major developments in hydrothermal and thermal system scheduling methods are documented in chapter 2. To date, the emphasis has been on thermal systems with extension to systems with a few hydro plants. There have been very few studies made of systems with only hydro generation or a significant proportion of hydro generation.

Water flow into storages and ponds and system real and reactive load are stochastic in nature, but for short term optimal control, inflows may be accurately considered deterministic. Although even short-term predictions of system load cannot be reliably made without great effort, the optimal control problem becomes far too complex if a stochastic solution is attempted. Therefore it is desirable to use a deterministic technique based on expected values of river flows and system loads, possibly followed by real-time schedule updating to compensate for forecast errors and unanticipated changes in system conditions.

1.1 HIERARCHIES IN POWER SYSTEM PLANNING, OPERATION AND CONTROL

In power system planning, operation and control, we must recognise two hierarchies of structure. One is in the time-domain and the other is both geographical and administrative.

Optimisation of a power system aims to minimise the cost of meeting energy requirements of the system in a manner consistent with reliable service. An interacting family of problems, each distinct in character, arises from optimisation over a range of time-spans which may be as short as a few minutes or as long as a decade

or more. The problems are instantaneous regulation, daily optimisation of the hour-by-hour schedule, medium term optimisation over a month or a week, annual optimisation and future planning over several years.

Future planning optimisation, looking at system expansion and modification, attempts to accurately consider the cost of capital works and interest charges, but must rely on estimates of fuel, labour and maintenance costs deduced from predicted power demand, outages, wages and fuel prices. The time span of future planning optimisation may itself be optimised, with load times varying with the nature and size of project.

On a shorter time scale, the annual optimisation problem, influenced by seasonal load and hydro-resource cycles, is responsible for week by week or month by month scheduling of long-term storage hydro plant, and for major maintenance scheduling. It assumes capital works, interest charges and administration costs have been determined in advance, with labour, maintenance and fuel costs estimated from predictions of demand, outages and fuel prices. An accurate system power flow model would be superfluous, and representation of losses as either a fixed quantity or a quadratic function of total system load would be adequate, since the instantaneous load demand and its distribution cannot be determined with any reasonable accuracy.

Medium term scheduling, often linked to the system weekly load cycle and daywork maintenance operations, is concerned with detailed scheduling of maintenance and refinement of the annual

resource-use schedule, serving to bridge the large gap in data accuracy available for annual and daily schedules. It is in medium term scheduling that river flow predictions are prominent.

The short term scheduling problem based on the system load cycle of one day, is concerned with unit commitment, hour-by-hour scheduling of both hydro and thermal plant, and system security and stability considerations. It requires an accurate and complete system power flow model and predictions of the magnitude and distribution of both active and reactive loads, but total water use at each hydro plant is largely preallocated by the medium and long term schedules.

In time spans of one hour or less, there is adjustment of the daily schedule to compensate for errors in load and stream flow predictions, and on-line control of frequency, voltage and line flows.

To obtain absolutely optimal operation of a power system the problems discussed above must be solved simultaneously as one. This is clearly impossible because of the dimension of the overall problem and because of the uncertainty of long term rainfall and load predictions, both of which are, and are likely to remain, very much an art rather than an exact science. Dividing the scheduling process into subproblems means that the overall solution is not strictly optimal, but will be near optimal, since each subproblem is solved optimally.

Operational planning and control of the power system is also a hierarchy in a geographical and functional sense. Tradition-

ionally the power system is divided into areas of generation, transmission and distribution, but in control and operation, a local, regional and central breakup of control functions is more useful. The local level often incorporates a degree of automatic control, such as governors, transformer tap controllers and automatic boiler controls. Regional centres control a larger part of the system, often carrying out load: frequency control and automatic voltage control. They may also exercise direct control over some power plants, as hydro power plants are now frequently unmanned. The problems of economic optimisation and security are usually the function of the central system control as are studies concerned with security, stability and longer term economic operation, which are usually conducted off-line. There is frequently an overlap of the functions of regional and system controllers.

1.2 SCOPE AND PLAN OF THESIS

This thesis concentrates on the solution of the short term, or daily scheduling of power systems having only hydrogeneration, or integrated power systems with a significant proportion of hydrogeneration. Unit commitment of hydro plant is included, but the unit commitment of thermal plant has not been considered as it is well covered elsewhere [14, 15]. Ideally, stream flows and system demand should be treated stochastically, but with a complete system model, the deterministic approach must be used to obtain acceptable computing times. As the solution scheme described generates sensitivity coefficients suited to very short term

schedule correction, we would expect this course to provide more accurate results than a "once only" stochastic solution.

The short-term schedule of a hydro or integrated power system is approached through a sequence of smaller problems in which alternately only hydro and only thermal or slack bus hydro plants are rescheduled, with reference to the same goal. The procedure refines an initial estimated hydro schedule, employing optimal load flow computations to schedule thermal or slack bus hydro generation, VAR allocation and regulator settings, alternating with a gradient or Newton search which uses the optimal load flow results including sensitivities to improve the hydro schedule. To make the subsequent refinement effective, the initial scheduling must determine unit commitment and bring the hydro schedule into a region where the costs are locally convex functions of the discharge rates, even though the generation vs discharge rate characteristics of multi-machine hydro plants have discontinuous slopes. The commonly used technique of step-loading is suitable. The loading points for each plant are the ends of the highest common tangent to sections of the generation vs discharge rate characteristic bracketing the average power required. A refinement of the usual manual step-loading method is presented, in which a global search is used. The dimensions of this search are reduced by reordering the chronological load curve into a load vs duration curve, and permitting only such combinations of steps in generator loading as are found to characterise optimal step-loading schedules on the reordered load curve.

By arranging the optimisation around one of the highly

developed optimal load flow algorithms devised for thermal power systems [4, 9, 10] we provide much of the sensitivity information required for planning, and also that needed to tell an operator not only the optimal schedule, but also the effects of small changes on it. As some optimal power flow algorithms generate both first and second order sensitivity information, a basis is also provided for an on-line schedule modification scheme.

The framework of the thesis is basically as set out above. Chapter 2 surveys previous work in optimal control of hydrothermal power systems and the historical basis of this work, reviewing conventional and quasi-conventional methods, together with some recent methods involving a substantially complete problem formulation. The methods are described and their relative merits and shortcomings are discussed.

In chapter 3, a mathematical model of an integrated power system, complete in those details relevant to short-term scheduling is formulated. It is adaptable to hydro or thermal power systems by deleting reference to the type of plant not present. Optimality criteria appropriate to short term scheduling of both hydro and integrated power systems were discussed, primarily with reference to total water use at each hydro plant except a slack bus hydro plant being pre-allocated by the medium term scheduling phase. Simplifications of the complete system model for the unit commitment process are also nominated.

A step loading technique for unit commitment in the hydro subsystem is developed in chapter 4. The object of this is to

provide a starting schedule for a later detailed scheduling process which ensures local convexity of the objective with respect to control variables by fixing unit commitment. Step-loading between two loads defined by a common tangent to a plant load vs discharge curve bracketing the average load required is shown to be optimal for plants in a system without transmission losses. Using these load steps, a method of step-loading based on reordering the chronological load curve to become a load vs duration curve is described. This produces a schedule which obeys water use constraints, minimises plant losses, and goes some way toward minimising transmission losses.

In chapter 5, the decomposition of a power system into an electrothermal subsystem solved separately in each time interval and a group of hydro subsystems, one for each river system is described, together with the resulting problem formulation. Four gradient and two second order, or Newton algorithms are devised for improvement of the hydro subsystem schedule, based on data provided by optimal load flow computations carried out for the electrothermal subsystem. The six hill-climbing algorithms are subjected to Monte-Carlo tests under simulated scheduling conditions, and assessed on their performance over a very few steps.

The problem of scheduling the electrothermal subsystem comprising the transmission system, regulators, thermal plants and reactive generation at hydro plants is discussed in chapter 6, with several methods which accurately represent this subsystem being studied.

Chapter 7 takes Dommel and Tinney's [4] method of scheduling the electrothermal subsystem, which is the method shown in chapter 6 to be the most practical for incorporation into an integrated scheduling scheme, and extensions to include area interchange optimisation [3] and automatic adjustment of tap changers and phase shifters [9, 10] within the Newton algorithm are made. A number of alternatives for handling of functional inequality constraints are discussed, considering both "soft" and "hard" limits.

In chapter 8, the decomposition system of chapter 5 is combined with the electrothermal subsystem scheduling method of chapter 7 to provide a scheme for short-term scheduling of a hydro or integrated power system. This schedule will obey all electrical network and water use constraints.

Sensitivity considerations in the integrated power system are discussed in chapter 9, primarily with reference to the analysis carried out by Peschon, Piercy, Tinney, and Tsviet [12]. Sensitivity of the schedule to changes on total discharge at each plant, i.e., to changes in the medium term schedule, are also discussed together with the practical implications and applications of the coefficients.

The features required of a scheduling scheme if it is to be accepted for use either off or on line in a real system are discussed in chapter 10. Particular consideration is given to the need for feasible schedules even if computation has to be cut short, and to the provision of sensitivity information for real-time schedule updating or for the guidance of system controllers. The thesis is reviewed and possible future extensions to the work are discussed including the potential application of the step loading technique

of chapter 4 to the medium term problem of water resource allocation.

Appendix 1 derives the optimal operating mode of a fixed head hydro plant using Pontryagin's maximum principle. It corrects a derivation previously attempted by Mantera [7, 8].

Appendix 2 documents the results of Monte-Carlo tests of hill climber methods discussed in chapter 4.

Appendix 3 contains copies of papers prepared in the course of the research.

1.3 REFERENCES

- [1] Carpentier, J. "Contribution a l'etude du dispatching economique" Bull. de la Soc. Fr. des Electriciens v4 (1963).

- [2] Carpentier, J. and Sirieux, J. "L'Optimisation de la production a Electricite de France" Bull. de la Soc. Fr. des Electriciens v4 (1963).

- [3] Britton, J.P. "Improved area interchange control for Newton's method load flows" Trans. IEEE paper 69 TP 124 presented at the 1969 power meeting.

- [4] Dommel, H.W. and Tinney, W.F. "Optimal power flow solutions", Trans. IEEE, Pwr. Appar. Sys. v89 (1968) pp 1866-1874.

- [5] Happ, H.H., Hohenstein, J.F., Kirchmayer, L.K., Stagg, G.W. "Direct Calculation of Transmission Loss Formula - II" Trans. IEEE Pwr. Appar. Sys. v83 (1964) pp702-707.

- [6] Kirchmayer, L.K., Happ, H.H., Stagg, G.W., Hohenstein, J.F. "Direct Calculation of Transmission Loss Formula - I" Trans. AIEE v79 (1960) III pp 962-969.

- [7] Mantera, I.G.M. "Optimal Daily Load Scheduling of Hydro Electric Systems by Means of Step Loading Technique." Unpublished.
- [8] Mantera, I.G.M. "Optimal Load Scheduling of Hydro Electric Power Stations" Ph. D. Thesis, Electrical Engineering Dept., University of Tasmania, June 1972.
- [9] Meyer, W.S. "New computational techniques for the power flow and economic dispatch problems of electric utility engineering". University of Minnesota, Ph. D., 1969.
- [10] Peterson, N.M. and Meyer, W.S. "Automatic adjustment of transformer and phase shifter taps in the Newton power flow Trans. IEEE Power meeting, Winter 1970.
- [11] Tinney, W.F., and Hart, C.E. "Power flow solution by Newton's Method". Trans. IEEE, Pwr. Appar. Sys. v86 (1967) pp 1449-1460.
- [12] Peschon, J., Piercy, D.S., Tinney, W.F., Tveit, O.J. "Sensitivity in power systems". Trans. IEEE Pwr. Appar. Sys. v87 (1968) pp 1687-1696.
- [13] Van Ness, J.E. and Griffin, J.H. "Elimination methods for load flow studies" Trans. AIEE Pwr. Appar. Sys. v80 (1961) pp 299-304.

- [14] Dillon, T.S. "Problems of optimal economic operation and control of integrated (hydro-thermal) and thermal power systems" Ph. D. Thesis, Department of Electrical Engineering, Monash University.
- [15] Lowery, P.G. "Generating unit commitment by dynamic programming" Trans. IEEE Pwr. Appar. Sys. v85 (1966) pp 422-426.

2. A HISTORICAL DEVELOPMENT OF OPTIMAL CONTROL IN HYDROTHERMAL POWER SYSTEMS

Use of hydro plants in an integrated hydrothermal power system to achieve optimal overall operation has attracted interest since the 1920's. It was realised then that run-of-river hydro plant was more efficiently used to supply the base load of a mixed system, with small storage schemes used for peaking, avoiding use of thermal plant at low efficiency output levels. Another favourite rule-of-thumb during this period [15] was to run each hydro plant at the maximum efficiency discharge rate whenever practical. These elementary methods were followed by the "equal incremental plant cost" method, paralleling a similar development in thermal economic control. This method assigned an artificial "cost" to water use by each hydro plant to enforce a total discharge constraint.

In their determination of generation schedules, these early methods did not incorporate the influence of transmission losses. An early investigation of transmission losses is that of [27], but the first practical representation was given by George [30], who modelled transmission losses as a function of source powers. Transmission loss

$$P_{TL} = \sum_{i=1}^n \sum_{j=1}^n P_{Gi} B_{ij} P_{Gj} \quad \dots \quad (2.1)$$

where P_{Gi} is the power generated by plant i

P_{Gj} is the power generated by plant j

B_{ij} are coefficients determined as indicated below

n is the number of generating plants

A number of methods presenting improved ways of computing the coefficients for this transmission loss formula were published in the 1950's. They included methods based on the network analyser [11, 30, 31, 36, 41], Kron's tensorial approach [42, 43], and some analogue and digital computer methods [35, 39]. Early et al. [24, 25] included linear and constant terms in their loss formula.

$$P_{TL} = \sum_{i=1}^n \sum_{j=1}^n P_{Gi} B_{ij} P_{Gj} + \sum_{i=1}^n B_{i0} P_{Gi} + B_{00} \dots (2.2)$$

This became known as the generalised loss formula and is still in common use for load dispatch.

The first significant advance towards co-ordination of incremental fuel rates and transmission losses in thermal systems was made by George et al. [31]. This was later extended to hydro-thermal systems [14], penalising each plant cost in accordance with the transmission losses incurred by generation at that plant. A similar approach considering incremental plant cost rates and transmission losses concurrently is due to Kirchmayer [38]. These are the "Equations of co-ordination" methods implemented on both analogue [23, 24] and digital [40] dispatching computers in the 1950's to optimise active generation. Transmission losses are a function of both active and reactive line flows, so optimisation of reactive VA generation is also necessary. Moskalev [49] extended the equations of co-ordination approach for active optimisation with a gradient method for reactive

optimisation.

In the 1960's, following the equations of co-ordination methods [33, 37, 38] which applied Lagrange multiplier theory to scheduling problems, more advanced optimal control techniques such as dynamic programming, [16, 29, 53] Pontryagin's maximum principle [2, 17, 34, 46, 48, 50] and linear programming [21] were applied to the variational problem of hydrothermal and hydro system optimisation.

The hydrothermal scheduling methods presented so far divide into two broad classes:

- (a) Conventional methods similar to those developed for thermal system dispatch. These use a simplified problem formulation in place of the complete formulation of chapter 3.
- (b) Quasi-conventional methods using advanced control techniques but operating on the same simplified problem formulation as that of the conventional methods.

These methods are characterised by lack of enforcement of constraints arising from network interconnection or equipment ratings with the occasional exception of limits on active generation. Transmission losses, when they are not neglected, are represented by approximations.

Several more recent approaches to scheduling are characterised by optimisation with respect to both active and reactive generation and by their ability to include all equipment ratings and network constraints. These were pioneered by Carpentier and Sirieux [12, 13] of Electricité de France, who developed the first complete

formulation of the problem of optimal control in thermal systems. Their use of nonlinear programming permitted inclusion of all equality and inequality constraints arising from system requirements and ratings. In most recent work, a.c. system conditions are represented by the network load flow model [56, 57]. The solutions obtained are not only optimal, but are always technically possible.

In some publications [1, 57], the optimal allocation of reactive generation has been handled on its own. This approach is suited to sub-optimal scheduling of a purely hydro power system.

The conventional methods are still in use by many utilities throughout the world. For this reason, and to allow proper evaluation of modern methods, a brief discussion of conventional and quasi-conventional methods for hydro-thermal system scheduling is presented. Discussion of conventional thermal dispatch methods is also included, as many methods for hydro-thermal system scheduling are adapted from them.

2.1 CONVENTIONAL APPROACHES TO THERMAL SYSTEM SCHEDULING

The primary methods of thermal system dispatch since the 1920's were [37, 38, 55]:

- (a) Base loading to capacity (the "merit-order" method)
- (b) Base loading to most efficient load
- (c) Loading proportional to capacity
- (d) Loading proportional to most efficient load
- (e) Equal incremental heat rate loading
- (f) Equal incremental cost rate loading, and
- (g) Methods using the equations of co-ordination.

These methods do not enforce network constraints or equipment ratings, and all but the last ignore dependence of transmission losses on the generation pattern. The equations of co-ordination method exists in several forms [22, 37], [38, 49], some taking reactive VA into consideration.

2.1.1 Merit order method [58]

If the incremental generating cost of any thermal unit is regarded as substantially independent of output as for turbines with throttle governing, and system constraints are disregarded, the most economic allocation of generation is to load each unit to capacity in ascending order of incremental costs. This method does not include transmission losses.

2.1.2 Equal incremental costs method [37, 38]

When incremental generation costs are not substantially constant, i.e. the cost is not a linear function of output power, we can use Lagrange multiplier theory to minimise the cost function:

$$F = \sum_{i=1}^{n_T} F_i(P_{Ti}) \quad \dots (2.3)$$

subject to the load-balance equality constraint

$$\sum_{i=1}^{n_T} P_{Ti} - P_D - C_L = 0 \quad \dots (2.4)$$

where P_{Ti} is the active generation of thermal unit i , P_D is the system load and the transmission losses, C_L are considered independent

of generation pattern. The optimum is formed by minimising the Lagrange function.

$$\begin{aligned} \$ = F(P_T) - \lambda \cdot \left\{ \sum_{i=1}^{n_T} P_{Ti} - P_D - C_L \right\} \quad \dots \quad (2.5) \end{aligned}$$

The necessary conditions for a minimum are therefore

$$\frac{d\$}{dP_{Ti}} = 0 \quad i = 1, \dots, n_T$$

or

$$\lambda = \frac{dF(P_{Ti})}{dP_{Ti}} \quad i = 1, \dots, n_T \quad \dots \quad (2.6)$$

Equations (2.4) and (2.6) must be satisfied at the optimum. Equation (2.6) implies that the most economic distribution of load between units is such that they are operated with equal incremental cost rates. The Lagrange multiplier, therefore represents the overall incremental cost of system generation if transmission losses are neglected.

2.1.3 Equations of co-ordination methods [33, 37, 38, 49]

Extending the equal incremental cost approach to include an approximation of transmission losses, the load balance constraint (2.4) becomes

$$\begin{aligned} \sum_{i=1}^n P_{Ti} - P_{TL} - P_D = 0 \quad \dots \quad (2.7) \end{aligned}$$

where P_{TL} is the system transmission loss. The optimum is obtained

by minimising $F(P)$ as defined by (2.3) subject to equality constraint (2.7). The Lagrange function is now

$$\begin{aligned} \lambda &= F(P) - \lambda \cdot \sum_{i=1}^n (P_{Ti} - P_{TL} - P_D) \quad \dots (2.8) \end{aligned}$$

and setting $\partial \lambda / \partial P_{Ti} = 0$ we obtain

$$\lambda = \frac{\partial F_i}{\partial P_{Ti}} / \left(1 - \frac{\partial P_{TL}}{\partial P_{Ti}}\right); \quad i = 1, \dots, n \quad \dots (2.9)$$

These are the "equations of co-ordination", and $\frac{\partial P_{TL}}{\partial P_{Ti}}$ is the

incremental transmission loss. It is usual to set

$$\lambda = \frac{\partial F_i}{\partial P_{Ti}} \left(1 + \frac{\partial P_{TL}}{\partial P_{Ti}}\right)$$

so that the incremental transmission loss is charged at the incremental cost rate, $\partial F(P) / \partial P_{Ti}$ rather than at the incremental cost of received load.

Moskalev [49] considered a cost function of the form

$$\begin{aligned} F &= F_T(P, Q) = \sum_{i=1}^n F_{Ti}(P_{Ti}, Q_{Ti}) \quad \dots (2.10) \end{aligned}$$

with equality constraints

$$W_1 = \sum P_{Ti} - \sum P_{Dj} - P_{TL} = 0 \quad \dots (2.11)$$

$$W_2 = \sum Q_{Ti} + \sum Q_{Cj} - \sum Q_{Dj} - Q_{TL} = 0 \quad \dots (2.12)$$

where P_{Ti} is the active generation by thermal plant at bus i ,

Q_{Ti} is the reactive generation by thermal plant at bus j ,

Q_{Cj} is the reactive generation by synchronous or static condensers at bus j ,

P_{Dj} is the active power demand at bus j ,

Q_{Dj} is the reactive VA demand at bus j ,

P_{TL} is the active power transmission loss, and

Q_{TL} is the reactive VA transmission loss.

The Lagrange function is

$$\Phi = F_T + \lambda_1 W_1 + \lambda_2 W_2 \quad \dots (2.13)$$

setting $\partial\Phi/\partial P_{Ti} = 0$ and $\partial\Phi/\partial Q_{Ti} = 0$ and noting that for modern power plants

$$\frac{\partial F_i}{\partial Q_i} \ll \frac{\partial F_i}{\partial P_i} \quad \dots (2.14)$$

Moskalev obtains

$$\lambda_1 = \frac{\partial F_i}{P_{Ti}} \left(1 - \frac{\partial Q_{TL}}{\partial Q_{Ti}}\right) / \left(1 - \frac{\partial P_{TL}}{\partial P_{Ti}} - \frac{\partial Q_{TL}}{\partial Q_{Ti}}\right) \quad \dots (2.15)$$

$$\lambda_2/\lambda_1 = \frac{\partial P_{TL}}{\partial Q_{Ti}} / \left(1 - \frac{\partial Q_{TL}}{\partial Q_{Ti}}\right) \quad \dots (2.16)$$

These two equations show that the problems of active and reactive generation scheduling are not separable in the search for the most economic schedule.

2.2 CONVENTIONAL APPROACHES TO HYDROTHERMAL SCHEDULING

The three most significant methods in the category are:

- (a) Maximum efficiency [15, 38]
- (b) Equal incremental cost rates [15, 38]

(c) The equations of co-ordination [38]

These methods of short-term optimisation of a hydro- or hydro-thermal power system use a simplified system model without network constraints or equipment ratings. They may therefore produce insecure or unstable solutions. Methods (b) and (c) are adapted from techniques devised originally for thermal systems, requiring assessment of an equivalent cost of water for each reservoir.

Normally a conversion coefficient γ_j for each hydro plant, j , converts the incremental water use rate, $\frac{\delta u_j}{\delta P_j}$ into an equivalent incremental plant cost rate.

2.2.1 Maximum efficiency method

This method dictates operation of hydro plants at efficiency peaks on their power vs. discharge rate characteristic except in times of surplus water, with thermal plants scheduled by merit order, equal incremental cost, or the co-ordination equations method discovered in 2.1. This is the general step-loading technique currently used by many utilities.

The step-loading technique proposed by Mantera [48], and extended as a hydro-unit-committment scheme in this thesis, is a maximum efficiency method which optimises the times of switching between two discharge rates, one above and one below the average scheduled discharge rate, close to maximum efficiency points on the power vs discharge rate curve at each plant.

2.2.2 Equal incremental costs method

Transmission losses are neglected, and the cost function

$$\begin{aligned} \sum_{i=1}^{n_T} \dot{\$}_i (P_{Ti}) + \sum_{j=1}^{n_H} \gamma_j u_j \end{aligned} \quad \dots \quad (2.17)$$

is minimised subject to the demand constraint

$$\sum_{i=1}^{n_T} P_{Ti} + \sum_{j=1}^{n_H} P_{Hj} - P_D = 0 \quad \dots \quad (2.18)$$

where P_{Ti} is the thermal generation at bus i ,

P_{Hj} is the hydro generation at bus j ,

P_D is the total system load,

u_j is the total discharge rate of hydro plant j .

Introducing a Lagrange multiplier, to attach the constraint (2.18) to the cost function

$$\frac{\partial \dot{\$}_i (P_{Ti})}{\partial P_{Ti}} = \lambda \quad \text{for } i = 1, 2, \dots, n_T \quad \dots \quad (2.19)$$

$$\gamma_j = \frac{\partial u_j}{\partial P_{Hj}} = \lambda \quad \text{for } j = 1, 2, \dots, n_H \quad \dots \quad (2.20)$$

where γ_j is a weighting factor adjusted to ensure that a predetermined volume, q_j , of water is discharged by hydro plant j over the whole scheduling time-span. The value of γ_j must be obtained iteratively, and both digital and analogue solution schemes have been devised.

2.2.3 The equations of co-ordination method

Also known as the Lagrange multiplier technique, this method was developed for hydrothermal systems by Kirchmayer [38] and extended by Chandler and Gabrielle [14] to cover all hydro systems.

Mantera [45,48] corrected a mathematical error in [14] which restricted it to transmission loss minimisation.

The method is similar to the equal incremental costs method. Equation (2.18) is replaced by

$$\sum_{i=1}^{n_T} P_{Ti} + \sum_{j=1}^{n_H} P_{Hj} - P_D - P_{TL} = 0 \quad \dots (2.21)$$

where P_{TL} is the system transmission loss. At the optimum [32, 38]

$$\frac{\partial \phi_i(P_{Ti})}{\partial P_{Ti}} + \lambda \cdot \frac{\partial P_{TL}}{\partial P_{Ti}} = \lambda \quad \dots (2.22)$$

$$\gamma_j \frac{\partial u_j}{\partial P_{Hj}} + \lambda \cdot \frac{\partial P_{TL}}{\partial P_{Hj}} = \lambda \quad \dots (2.23)$$

The incremental transmission losses are usually obtained from a quadratic loss formula such as that proposed by Early et al. [24, 25] (equation 2.2).

Head variations can be incorporated by allowing to vary [32, 38]

$$\gamma_j = \gamma_{j0} \exp \left(\int_0^T \frac{1}{A_j} \left[\frac{\partial u_j}{\partial H_j} \right] P_{Hj} = \text{const.} \cdot dt \right) \quad \dots (2.24)$$

where u_j and A_j are the discharge rate and surface area of reservoir

j , and $\left[\frac{\partial u_j}{\partial H_j} \right] (P_{Hj} = \text{const.})$ is the rate of change or discharge rate

with head for a fixed generation at plant j .

2.2.4 A practical comparison of conventional methods for hydrothermal system scheduling

A computational comparison of the three methods above, undertaken by Kirchmayer [38], found that, except for low water use, when all methods yielded similar results, the ascending order of performance was:

- (i) Maximum efficiency hydro method with equal incremental costs thermal generation (2.2.1 and 2.1.2)
- (ii) Equal incremental costs method (2.2.2)
- (iii) Maximum efficiency hydro method with co-ordinated thermal generation (2.2.1 and 2.1.3)
- (iv) The equations of co-ordination method (2.2.3)

Kirchmayer's results are illustrated in Fig. 2.1 for a system comprising three independent hydro plants and one thermal plant.

2.3 QUASI-CONVENTIONAL APPROACHES TO HYDROTHERMAL SCHEDULING

Using variational techniques including linear programming [21, 28], dynamic programming [16, 48, 53] and Pontryagin's maximum principle [2, 17, 34, 46, 48, 50], these methods solve the simplified hydrothermal scheduling problem. Transmission losses are neglected or treated by an approximate loss formula. Equipment ratings and network constraints are also disregarded except for limits on hydro discharge rates. As with conventional approaches, unstable or insecure solutions may result.

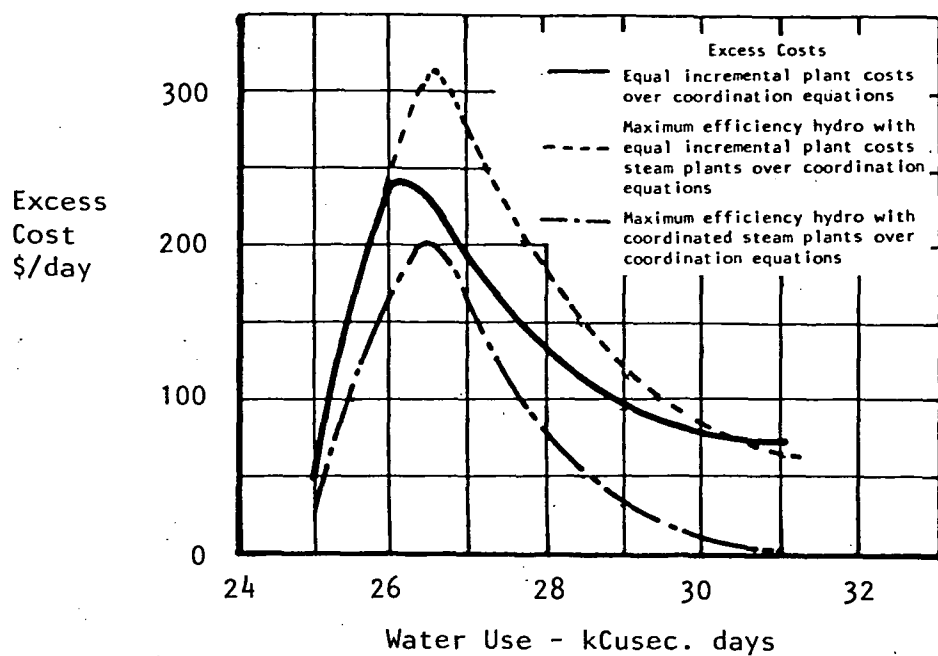


Fig 2.1 [38] Excess costs of equal incremental plant costs scheduling and maximum efficiency hydro scheduling over coordination-equations scheduling

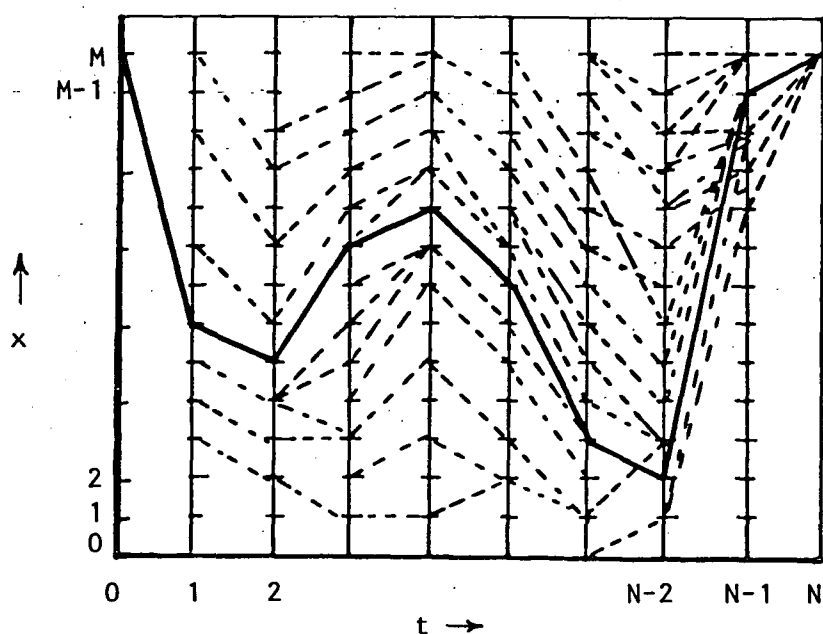


Fig 2.2 [18] The dynamic programming procedure

2.3.1 Linear programming [21, 28]

The problem is transformed into one of linear programming by discretising in the time domain, setting up state differential equations in discrete form as equality constraints and replacing the integral cost functional by a summation. The curve of power vs discharge rate for each plant are linearised and transmission losses are neglected.

In the scheme developed by Di-Perna and Ferrara [21] the optimisation objective is to maximise the total energy produced by all hydro plants over the scheduling time-span, subject to total water use constraints on each plant.

2.3.2 Dynamic programming

The method is based on Bellman's principle of optimality, which, in control terms, may be stated [26]:

"A policy which is optimal over the interval $[0, m]$
is necessarily optimal over any subinterval $[v, m]$
where $0 \leq v \leq m - 1$ "

This principle does not apply to all systems. In particular, some systems with integral constraints do not obey it.

For a one-state-dimensional example, i.e. one hydro storage with time and state discretised into k and ℓ subdivisions respectively, the procedure is first to find the optimal path from each of the ℓ states (grid points) at time $t = k - 2$ to the endpoint using Bellman's principle of optimality. We proceed in this manner until we arrive at time $t = 0$. The initial state is known, so the optimal trajectory may be retraced. The procedure is illustrated in

Fig. 2.2.

The number of grid points expands with the number of hydro plants. In a system with n_H hydro plants, this may be as high as $k \cdot n_H - 1$ rendering the technique impractical for all but the smallest system. This difficulty may be reduced by a successive approximations approach as used by Bernholtz and Graham [5], Fukao, Yamazaki and Kimura [29], Dahlin and Shen [16] and Mantera [48].

Two of these researchers [5, 48] also proposed that, starting from a feasible schedule, only one plant schedule should be relaxed per successive approximation iteration, and that only perturbations of $\pm \Delta P_{Gi}$ from the previous generation schedule of that plant be permitted at each iteration. The result is an increased number of iterations, but a great reduction in the storage requirement and number of operations per iteration over the methods proposed in [16] and [29]. The number of iterations required is very sensitive to the initial schedule.

The "state increment dynamic programming" technique developed by R.E. Larson [44] uses a time increment chosen as the smallest required for at least one state variable to change by one increment. If applied to power system scheduling, this technique may result in a reduction in the number of grid points, but a more significant benefit lies in the increase in accuracy which could result from application of this technique to the successive approximations approach.

2.3.3. Pontryagin's maximum principle

Applied to the hydrothermal scheduling problem by Hano,

Tamura and Narita [34], Dahlin and Shen [17], Oh [50], Belev [2] and Mantera [48], this approach was a simplified problem formulation in which transmission losses are represented by loss differentials with respect to active power generated at each bus, and inequality constraints and equipment ratings are ignored.

We set up a Hamiltonian

$$H = -\dot{\$eq}(P_T) + \sum_{i=1}^{n_H} p_i \cdot f_i \quad \dots (2.25)$$

in which $\dot{\$eq}(P_T)$ is the cost rate of thermal generation P_T , $\{p_i\}$ is the set of adjoint variables given by

$$\dot{p}_i = \frac{\partial H}{\partial x_i} \quad \dots (2.26)$$

and f_i is the right hand side of the state differential equation

$$\dot{x}_i = I_i - u_i = f_i \quad \dots (2.27)$$

Here I_i and u_i are the inflow and discharge rates of hydro storage i .

The condition for the optimum of cost is the maximisation of the Hamiltonian H over the admissible control space. For this we require $p_i(t)$ specified by differential equation (2.21). Since the hydro optimisation is a two point boundary value problem, this is usually obtained by estimating $p_i(t_0)$ integrating to final time T while maximising H , and noting the endpoint error. A Newton-Raphson step is then performed to improve the estimate of $p_i(t_0)$. It is noted that the adjoint equations are often unstable in the forward direction, but this approach is usually adopted as it is far simpler when it is stable, having modest storage requirements and being well-

suited to analogue or hybrid computation.

2.4 WEAKNESSES OF CONVENTIONAL AND QUASI-CONVENTIONAL OPTIMISATION METHODS IN HYDROTHERMAL SYSTEMS

Both conventional and quasi-conventional methods for scheduling of active generation in hydrothermal systems suffer many shortcomings, the most serious being that

- (a) Transmission losses may be incorporated only as loss-differentials with respect to active power generation at each bus. This requires that the network be separable into much simpler networks or satisfy the assumptions made in derivation of B-coefficients [42, 43]. Many early methods ignore transmission losses completely.
- (b) Except in the work of Moskalev [49], optimisation with respect to active generation is not carried out. This limitation arises from the B-coefficient loss representation.
- (c) With the exception of limits on active generation, no attempt is made to take into account equipment ratings or security and stability limits on lines and cables.
- (d) Constraints on hydro plant discharge rates, in practice frequently state- and time-dependent, are either fixed or ignored.

In many small "concentrated" systems, conventional or quasi-conventional methods are sufficient, as the assumptions involved in B-coefficient calculation are approximately valid and there are few

stability problems. In these cases, reactive VA generation is allocated to keep all equipment operating within its ratings.

With the increasing interconnection of power systems and proliferation of small-capacity but wide-spread systems, the conventional and quasi-conventional methods are no longer adequate, since B-coefficients cannot accurately model transmission losses, and non-enforcement of equipment ratings may lead to unrealisable, insecure or unstable schedules.

2.5 HYDROTHERMAL SCHEDULING BASED ON AN ACCURATE SYSTEM MODEL

The use of optimization methods for an accurate system model of a purely thermal power system was pioneered by Carpentier and Sirieux [12, 13]. The methods are characterized by optimization with respect to both active and reactive generation, and by the ability to include all equipment ratings and equality and inequality constraints describing the transmission networks. Such schemes, which almost without exception utilise the a.c. network load flow model to describe system conditions, are demanding of computer time and storage, but can produce solutions which are always stable and realisable. Use of the a.c. network load flow model enables these methods to provide sensitivity data useful in system planning. A selection of these methods is summarised below.

2.5.1 Nonlinear programming with decomposition

This method, foreshadowed by Bernholtz and Graham [5-8] was later realised by successive approximations using dynamic programming by Bonaert, El-Abiad and Koivo [9, 10] and by gradient and second-order methods in this thesis. It is based on the problem formulation of chapter 3. Replacing the hydrothermal state-differ-

ential equations by discrete-time equivalents, the dynamic optimal control problem can be approximated by a static nonlinear programming problem. If, however, the optimal control problem has l state and n control variables, and m subintervals of time are used, there will be at least $m \cdot (l + n)$ variables in the nonlinear programming problem, too many to be handled simultaneously by most computers if the power system is large.

To avoid this difficulty, the nonlinear programming problem is decomposed into m static electrothermal subsystems each representing conditions in one time interval, and l dynamic hydro subsystems each representing a river system, then co-ordinated to give the solution of the integrated system. This decomposition is detailed in chapter 5.

2.5.2 An initial value method based on a generalised maximum principle

Similar to that of Carpentier and Sirieux [12, 13] this method was developed by Dillon and Morsztyn [18, 19, 20] and requires a powerful computer for implementation.

Pontryagin's maximum principle [52] is valid for optimal control of systems for which the controls form a fixed, closed set. Since the control constraints are state-dependent, more general conditions derived by Berkovitz [3, 4] are used. For a system with n_T thermal plants and n_H hydro plants, of which the first n_H form a cascade, the Hamiltonian is

$$H(\underline{x}, \underline{u}, t) = \sum_{i=1}^{n_T + n_H + 1} p_i \cdot f(\underline{x}, \underline{u}, t)$$

$$= \sum_{i=1}^{n_T} p_i \cdot S(p_{Ti}, t) + \sum_{i=1}^{n_H} p_i + n_T \cdot (I_i(t) - u_i(t) + \sum_{j=1}^{n_H} u_j(t)) \quad (11)$$

$$+ \sum_{i=n_H+1}^{n_T} p_i + n_T \cdot (I_i(t) - u_i(t))$$

... (2.28)

where hydro plants $\{i1\}$ are the first upstream neighbours of plant i . Writing $\mathcal{H} = -H + \underline{\mu}^T \cdot \underline{G}$ where \underline{G} are the equality and inequality constraints defining the admissible control space, with corresponding multipliers $\underline{\mu}$, the adjoint variables $\{p_i\}$ are given by

$$\dot{p}_i(t) = - \frac{\partial H}{\partial x_i} + \underline{\mu}^T \cdot \underline{G}(x_i) = \frac{\partial \mathcal{H}}{\partial x_i} \quad \dots (2.29)$$

Solving for $p_i(t)$ and noting that for all thermal plants $p_i = 0$, the transversality condition gives

$$p_i(t) = 1; \quad i = 1, \dots, n_T \quad \dots (2.30)$$

and (2.28) becomes

$$H(\underline{x}, \underline{u}, t) = - \sum_{i=1}^{n_T} S_i(p_{Ti}, t) + \sum_{i=1}^{n_H} p_i + n_T (I_i(t) + \sum_{j=1}^{n_H} u_j(t) - u_i(t)) \quad (11)$$

$$+ \sum_{i=n_H+1}^{n_T} p_i + n_T \cdot (I_i(t) - u_i(t))$$

... (2.31)

The optimal trajectory, \underline{x}^* is defined by

$$H^*(\underline{x}^*, \underline{u}^*, t) = \sup_{\underline{u}} H(\underline{x}, \underline{u}, t) \quad \dots (2.32)$$

The equations representing satisfaction of conditions (2.32) are

$$-\frac{\partial H}{\partial \underline{u}} + \mu \frac{\partial G}{\partial \underline{u}} = \frac{\partial \mathcal{H}}{\partial \underline{u}} = 0 \quad \dots (2.33)$$

$$\mu_i \cdot G_i = 0 \quad \dots (2.34)$$

and $\mu_i > 0$ if constraint $G_i < 0$ is not active

$\mu_i = 0$ if constraint $G_i < 0$ is active

Writing (2.33) in full, a set of optimisation equations for a hydrothermal power system is obtained [34], together with a set of exclusion equations (2.34). To obtain the supremum of the Hamiltonian over the admissible control space, the value of $p_i(t)$ over the whole time interval is required. This is given by differential equation (2.29) and hence if its initial value is known, and if it is integrable, the problem is solved. $p_i(0)$ is not given by boundary conditions in this two-point boundary value problem, and can be found only by a trial and error process such as Newton-Raphson or Gauss-Levenberg search procedure [3].

2.5.3 A conjugate gradients method based on the generalised maximum principle [18]

This method is based on the above analysis. Equation (2.33) is first relaxed while meeting all other conditions, then it is iteratively approached using a conjugate gradient step to modify

the control variables. The endpoint conditions are enforced using penalty functions with variable weighting.

2.6 SUMMARY

Early attempts to efficiently control integrated power systems primarily used extensions of conventional methods for thermal dispatch. The conventional and quasi-conventional methods detailed in sections 2.2 and 2.3, whilst computationally simpler than recent methods, suffered from several serious shortcomings. When used, their results must be interpreted with caution.

The solution methods discussed in 2.5 always give physically realisable, stable and secure solutions when allowed to converge fully. They also take transmission losses into account directly as they employ the complete a.c. network load-flow representation. Sensitivity information useful in instantaneous system dispatch, a rescheduling for unexpected load changes or river flow changes, and for system growth planning is also generated. The methods described in 2.5.2 and 2.5.3 do not necessarily produce an improved schedule, nor do they guarantee that the schedule obtained will be physically realisable, stable or secure if computations are halted before convergence. The decomposition methods noted in 2.5.1 and applied using gradient, and second-order methods in this thesis is able to have solution halted at any stage to provide an improved, realisable, stable and secure schedule. This is useful, particularly when using a small, slow computer system. It also allows a hands-on approach to practical implementation which should facilitate acceptance of the technique, as well as allowing partial implementation for a system with inadequate telemetering.

2.7 REFERENCES

- [1] Baumann, R. "Power Flow Solution with Optimal Reactive Flow". Archiv. fur Electrotech. 48 (1963) 213-244.
- [2] Beleav, L.S. "Application of Pontryagin's maximum principle for the Optimisation of the Operation of Complex Electrical Power Systems". Energetica & Transport No. 5 (1965) 13-22.
- [3] Berkovitz, L.D. "Variational Methods in Problems of Control and Programming". J. Math. Analysis Applic. 3 (1961) 145-169.
- [4] Berkovitz, L.D. "On Control Problems with Bounded State Variables". J. Math. Analysis Applic. 5 (1962) 488-498.
- [5] Bernholtz, B., Graham, L.J. "Hydrothermal Economic Scheduling, Part I: Solution by Incremental Dynamic Programming". Trans. AIEE 79 (1960) III, 921-932.
- [6] Bernholtz, B., Graham, L.J. "Hydrothermal Economic Scheduling, Part II: Extension of Basic Theory". Trans. AIEE 80 (1961) III, 1089-1096.
- [7] Bernholtz, B., Graham, L.J. "Hydrothermal Economic Scheduling, Part III: Scheduling the Thermal Subsystem Using Constrained Steepest Descent". Trans. AIEE 80 (1961) III, 1096-1105.

- [8] Bernholtz, B., Graham, L.J. "Hydrothermal Economic Scheduling, Part IV: A Continuous Procedure for Minimising the Weighted Output of a Hydro-Electric Generating Station". Trans. AIEE 80 (1961) III, 1105-1107).
- [9] Bonaert, A.P., El-Abiad, A.H., Koivo, A.J. On the Optimal Scheduling of Hydrothermal Power Systems TR-EE 70-21, School of Electrical Engineering, Purdue University, March 1971.
- [10] Bonaert, A.P., El-Abiad, A.H., Koivo, A.J. "Optimal Scheduling of Hydro-Thermal Power Systems". Trans. IEEE PAS 91 (1972) 2, 1412-1419.
- [11] Brownlee, W.R. "Co-ordination of Incremental Fuel Costs and Incremental Transmission Losses by Functions of Voltage-Phase Angles". Trans. AIEE 73 (1954) III A, 529-541.
- [12] Carpentier, J. "Contribution of l'Etude du Dispatching Economique" Bull. de la Soc. Fr. des Electriciens 4 (1962) 431-447.
- [13] Carpentier, J., Sirieux, J. "L'Optimization de la Production a Electricite de France". Bull. de la Soc. Fr. des Elect-riciens 4 (1963) 121-129.
- [14] Chandler Jr., W.M., Gabrielle, A.F. "Economy Loading of Hydro-Electric Systems". M.Sc. Thesis, M.I.T., 1950.

- [15] Creager, W.P., Justin, J.D. Hydro-Electric Handbook.
2nd Ed., New York: Wiley, 1950.

- [16] Dahlin, E.B., Shen, D.W.C. "Application of Dynamic
Programming to Optimisation of Hydroelectric/Steam Power
System Operation". Proc. IEE (Power), 112 (1965) 2255-2260.

- [17] Dahlin, E.B., Shen, D.W.C. "Optimal Solution to the Hydro-
Steam Dispatch Problem for Certain Practical Systems".
Trans. IEEE PAS 85 (1966) 437-458.

- [18] Dillon, T.S. "Problems of Optimal Economic Operation and
Control of Integrated (Hydrothermal) and Thermal Power
Systems" Ph.D. Thesis, Electrical Engineering Department,
Monash University, April 1974.

- [19] Dillon, T.S., Morsztyn, K. "Optimal Control of Integrated
Power Systems". Report from the Electrical Engineering
Department, Monash University, MEE 69-3 (1969).

- [20] Dillon, T.S., Morsztyn, K. "A New Theoretical and Comp-
utational Approach to the Exact Solution of the Problem of
Optimal Control of Integrated (Hydro-Thermal) Power Systems"
5th. IFAC World Congress, Paris, June 1972.

- [21] Di-Perna, A., Ferrara, E. "Optimal Scheduling of a Hydro-
Electric System by Linear Programming" Power System

Computation Conference, Sweden, 1966. Report 6.14.

- [22] Dopazo, J.F., Klitin, D.A., Stagg, G.W., Watson, M. "An Optimisation Technique for Real and Reactive Power Allocation". Power Industry Computer Applications Conference Record, Pittsburgh (1967) 141-154.

- [23] Early, E.D., Phillips, W.E., Shreve, W.T. "An Incremental Cost of Power Delivered Computer". Trans. AIEE 74 (1955) III, 592-535.

- [24] Early, E.D., Watson, R.E. "A New Method of Determining Constants for the General Transmission Loss Equation". Trans. AIEE 75 (1965) III, 1417-1423.

- [25] Early, E.D., Watson, R.E., Smith, G.L. "A General Transmission Loss Equation". Trans. AIEE 74 (1955) III, 510-517.

- [26] Elgerd, O.I. Control System Theory New York: McGraw-Hill 1967.

- [27] Estrada, H., Dryan, H.A. "Determining Tie-line Losses" Electrical World, October 1931.

- [28] Fanshel, S., Lynes, E.S. "Economic Power Generation Using Linear Programming". Trans. IEEE PAS 83 (1964) 347-356.

- [29] Fukao, T., Yamazaki, T., Kimura, S.. "An Application of Dynamic Programming to Economic Operation Problem of a Power System" Electrotechnical J. of Japan 5, No. 2 (1959) 64-68.
- [30] George, E.E. "Intrasystem Transmission Losses" Trans. AIEE 62 (1944) 153-158.
- [31] George, E.E., Page H.W., Ward, J.B. "Consideration of Fuel Cost and Transmission Loss by Use of the Network Analyser to Determine Plant Loading Schedules". Trans. AIEE 68 (1949) II 1152-1160.
- [32] Glimn, A.F., Kirchmayer, L.K. "Economic Operation of Variable-Head Hydro-Electric Plants" Trans. AIEE 77 (1958) III, 1070-1073.
- [33] Glimn, A.F., Kirchmayer, L.K., Haberman Jr., R., Thomas, R.W.. "Automatic Digital Computer Applied to Generation Scheduling". Trans. AIEE 73 (1964) IIIB, 1267-1275.
- [34] Hano, I., Tamura, Y., Narita, S. "An Application of the Maximum Principle to the Most Economical Operation of Power Systems". Trans. IEEE PAS 85 (1966) 486-494.
- [35] Happ, H.H., Hohenstein, J.F., Kirchmayer, L.K., Stagg, G.W. "Direct Calculation of Transmission Loss Formula - III". Trans. IEEE PAS 83 (1964) 702-707.

- [36] Harder, E.L., Ferguson, R.W., Jacobs, W.E., Harker, D.C.
"Loss Equation - Part II, Current and Power Form Loss
Formulas". Trans. AIEE 73 (1954) 111A, 716-731.
- [37] Kirchmayer, L.K. Economic Operation of Power Systems
New York: Wiley, 1959.
- [38] Kirchmayer, L.K. Economic Control of Interconnected
Systems. New York: Wiley, 1959.
- [39] Kirchmayer, L.K., Happ, H.H., Stagg, G.W., Hohenstein, J.F.
"Direct Calculation of Transmission Loss Formula - I"
Trans. AIEE 79 (1960) III 962-969.
- [40] Kirchmayer, L.K., McDaniel, G.H. "Transmission Losses and
Economic Loading of Power Systems". General Electric
Review, 54 (1961) 1152-1163..
- [41] Kirchmayer, L.K., Stagg, G.W.. "Analysis of Total and
Incremental Losses in Transmission Systems". Trans. AIEE
70 (1951) I, 1197-1204.
- [42] Kron, G. "Tensorial Analysis of Integrated Transmission
Systems - Pt. I: The Six Basic Reference Frames". Trans.
AIEE 70 (1951) II, 1239-1248.

- [43] Kron, G. "Tensorial Analysis of Integrated Transmission Systems - Pt. II: Off-Nominal Turns Ratios" Trans. AIEE 71 (1952) P1 505-512.
- [44] Larson, R.E. "Dynamic Programming with Reduced Computational Requiriements". Trans. IEEE Auto. Control 10 (1965) 135-143.
- [45] Mantera, I.G.M. "Optimum Hydro Electric Power Generation Scheduling by Analogue Computer". Proc. IEE 118 (1971) 110-116.
- [46] Mantera, I.G.M. "Optimal Daily Load Scheduling of Hydro Electric Systems by Means of Step Loading Technique, Part I" Unpublished.
- [47] Mantera, I.G.M. "Optimal Daily Load Scheduling of Hydro Electric Systems by Means of Step Loading Technique, Part II" Unpublished.
- [48] Mantera, I.G.M. "Optimal Load Scheduling of Hydro Electric Power Stations" Ph.D. Thesis, Electrical Engineering Dept., University of Tasmania, June 1972.
- [49] Moskalev, A.G. "Principles of the Most Economic Distribution of the Active and Reactive Loads in the Automatically Controlled Power Systems". Elektrichestvo 24 No. 12 (1963) 24-33.

- [50] Oh, Y.N. "An Application of the Discrete Maximum Principle to the Most Economical Power System Operations" Electrical Engineering in Japan 85 (1964) 23-33.
- [51] Peschon, J., Piercy, D.S., Tinney, W.G., Tveit, O.J., Cuenod, M. "Optimum Control of Reactive Power Flow" Trans. IEEE PAS 87 (1968) 40-48.
- [52] Pontryagin, L.S., Boltyanskii, V.G., Gamkrelidze, R.V., Mishenko, E.F. The Mathematical Theory of Optimal Processes New York: Interscience Publishers, 1962.
- [53] Peterson, H. "An Example of the Use of Dynamic Programming for Economic Operation of a Hydro-Thermal System" Power System Computation Conference, Sweden 1966, Report 6.4.
- [54] Squires, R.B., Colburn, H.W., Byerly, R.T., Hamilton, W.R. "Loss Evaluation - Part V: Economic Dispatch Computer Design" Trans. AIEE 75 (1956) 719-727.
- [55] Steinberg, M.J., Smith, T.H. Economy Loading of Power Plants and Electric Systems New York: Wiley 1943.
- [56] Tinney, W.F., Hart, G.E. "Power Flow Solution by Newton's Method" Trans. IEEE PAS 86 (1967) 1449-1460.

- [57] Van Ness, J.E., Griffin, J.H. "Elimination Methods for Load Flow Studies" Trans. AIEE 80 (1961) III 299-304.
- [58] Central Electricity Generating Board of Great Britain
Modern Power Station Practice Vol. 7 (Operation and Efficiency), Pergamon Press, 1971 pp 297-300.

3. A MATHEMATICAL MODEL OF THE HYDROTHERMAL POWER SYSTEM FOR SHORT-TERM OPTIMISATION

Many of the shortcomings of early power system scheduling techniques result from restrictive and incomplete problem formulation (Chapter 2.). An adequate problem formulation must include an appropriate optimality criterion, definition of a comprehensive set of state and control variables, and mathematical expression of system equality and inequality constraints in terms of these variables. Typical equality constraints on the systems are the power flow equations and the hydro and thermal plant characteristics.

Stream flows and system demand cannot be accurately known in advance and therefore should ideally be treated stochastically. With a complete system model, the deterministic approach must be used to obtain acceptable computing times. As described in chapter 9, the proposed solution method produces sensitivity coefficients suitable for very short-term schedule correction. We would expect this to provide more accurate results than a "once only" stochastic solution. Alternatively, the deterministic solution may provide the basis for a Monte Carlo approach solving the stochastic problem, but as Dillon [5] noted, the computing times involved are extremely large.

3.1 THE OPTIMALITY CRITERION

A criterion for optimal short-term scheduling of a hydro-thermal power system requires that either reservoir depletion at all hydro plants be specified in advance by a longer term schedule [3], or

that a value be assigned to water [10] used where reservoir depletion is not specified. This value is based on two factors: one, reflecting the future value of water, depends on the medium and long-term stream flow and power demands. The other, indicating the short-term cost of supplying power from alternate sources, can be calculated for any plant at any time as a Lagrange multiplier in the course of optimising the thermal schedule with hydrogeneration temporarily fixed, using the scheme proposed in this thesis. To avoid encroaching on the longer-term scheduling problem we shall consider the optimality criterion to be minimisation of thermal plant fuel cost with specified reservoir depletion [3].

With specified reservoir depletion, a hydro system with tielines to adjoining systems would aim to minimise the cost of power purchases while operating within the supply contract [13]. If this contract was for supply at certain hours only, it may be necessary to nominate a plant within the system to take up variations in load. An isolated system composed wholly of hydro plants might use the criterion of meeting power demands with minimum depletion from its smaller pondages. In this case, specified large storage hydro plants with no short-term constraints on total water use would provide "slack-bus" power and be scheduled in the same way as thermal plant.

The above criteria are subject to ensuring reliability of supply, statutory limitations, and satisfactory quality of supply as characterised by voltage and frequency.

3.2 SELECTION OF CONTROL VARIABLES

As noted in Chapter 2, restriction of the set of control variables for an integrated power system to

$D_{Hj}(t)$, the useful discharge from hydro plants, and

$P_{Ti}(t)$, the real power generation of the thermal plants

limits representation of transmission losses to incremental transmission loss methods. As noted in 2.4, this is unsatisfactory, particularly in small capacity and widespread systems. Using only these control variables, it is not possible to take into account all constraints on the system, nor is it possible to optimise with respect to reactive VA.

To avoid such disadvantages, a more general set of control variables is chosen, not all of which are independent.

Useful discharge rate	$D_{Hj}(t)$ for all hydro storages or ponds
Active power generation	$P_{Gi}(t)$ for all power plants
Reactive VA generation	$Q_{Gi}(t)$ for all power plants and synchronous compensators
Voltage magnitude	$E_i(t)$ for every node on the system
Voltage phase angle	$\phi_i(t)$ for every node on the system
Transformer tap ratios	$\rho_{ij}(t)$ for all on-load tap-changing transformers (OLTC's)
Phase shifter setting	$\tau_{ij}(t)$ for all phase shifting transformers
Phase shifter power flow	$P_{ij}(t)$ for all phase shifting transformers

These control variables are related by inequality constraints, plant characteristics and load-flow equations, so only a selection can be

used as independent controls in any given case.

3.3.1 The load-balance constraints

At each node in the system, the power injected into the remainder of the system must equal the difference between power generated and power consumed at that node, and similarly, the reactive VA injected into the system at each node must equal the difference between the reactive VA generated and consumed at that node. These solutions are usually expressed in the form of the static load flow equations (SLFE).

A balanced three phase system is assumed, and the transmission system is represented by its positive-sequence network of lumped equivalent- π approximations to line and transformer characteristics. Consider a line containing no off-nominal ratio transformers or phase-shifters. The equivalent- π representation of Fig 3.1 is appropriate. The SLFE are

$$P_{ik}(\phi_{ik}, E_{ik}) = P_{Gik} - P_{Dik} = \sum_a \frac{E_{ik} E_{ak}}{Z_{ia}} \cdot \sin(\phi_{ik} - \phi_{ak} - \delta_{ia}) + \sum_a \frac{E_{ik}^2 \sin \delta_{ia}}{Z_{ia}} \quad \dots (3.1)$$

$$Q_{ik}(\phi_{ik}, E_{ik}) = Q_{Gik} - Q_{Dik} = - \sum_a \frac{E_{ik} E_{ak}}{Z_{ia}} \cdot \cos(\phi_{ik} - \phi_{ak} - \delta_{ia}) + \sum_a \frac{E_{ik}^2 \cos \delta_{ia}}{Z_{ia}} - Y_{ii} E_{ik}^2 \quad \dots (3.2)$$

where

P_{ik} is the total power generated	} at bus i in time interval k
P_{Dik} is the power demand	
Q_{Gik} is the reactive VA generated	
Q_{Dik} is the reactive VA demand	
$P_{ik}(\phi_{ik}, E_{ik})$ is the power injected	
$Q_{ik}(\phi_{ik}, E_{ik})$ is the reactive VA injected	
E_{ik} is the voltage magnitude	
ϕ_{ik} is the voltage phase angle relative to the slack bus	

and Z_{ia} , δ_{ia} , Y_{ia} are explained by Fig 3.1.

In the presence of off-nominal-ratio transformers or phase-shifters, these equations must be modified. Lines connected to any bus i may be divided into three categories illustrated by Fig 3.2. Line ia has no off-nominal ratio transformers or phase shifters, while ib and ic have a transformer or phase shifter at buses i and c respectively. Line ib may be represented by the equivalent circuit of Fig 3.3. The static load flow equations become as follows

$$\begin{aligned}
 P_{ik}(E_{ik}, \phi_{ik}, \rho_{ijk}, \tau_{ijk}) &= P_{Gik} - P_{Dik} \\
 &= \sum_a \frac{E_{ik} E_{ak}}{Z_{ia}} \cdot \sin(\phi_{ik} - \phi_{ak} - \delta_{ia}) + \sum_a \frac{E_{ik}^2}{Z_{ia}} \cdot \sin \delta_{ia} \\
 &\quad + \sum_b \rho_{ibk} \cdot \frac{E_{ik} E_{bk}}{Z_{ib}} \cdot \sin(\phi_{ik} - \phi_{bk} - \delta_{ib} + \tau_{ibk}) + \sum_b \rho_{ibk}^2 \frac{E_{ik}^2}{Z_{ib}} \cdot \sin \delta_{ib} \\
 &\quad + \sum_c \rho_{ick} \cdot \frac{E_{ik} E_{ck}}{Z_{ic}} \cdot \sin(\phi_{ik} - \phi_{ck} - \delta_{ic} - \tau_{ick}) + \sum_c \frac{E_{ik}^2}{Z_{ic}} \sin \delta_{ic} \\
 i &= 1, 2, \dots, n \qquad \dots (3.3)
 \end{aligned}$$

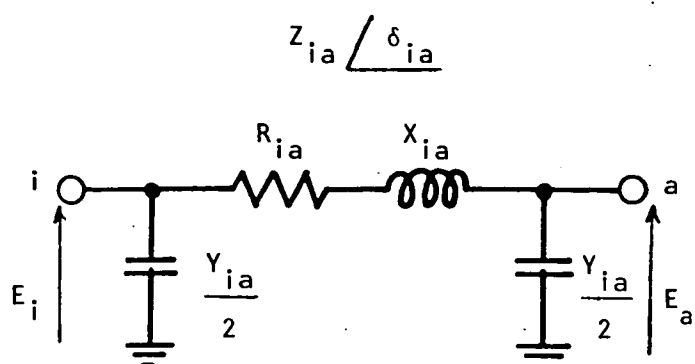


Fig 3.1 Equivalent - π Circuit approximation for a transmission line containing no off-nominal transformers or phase-shifters

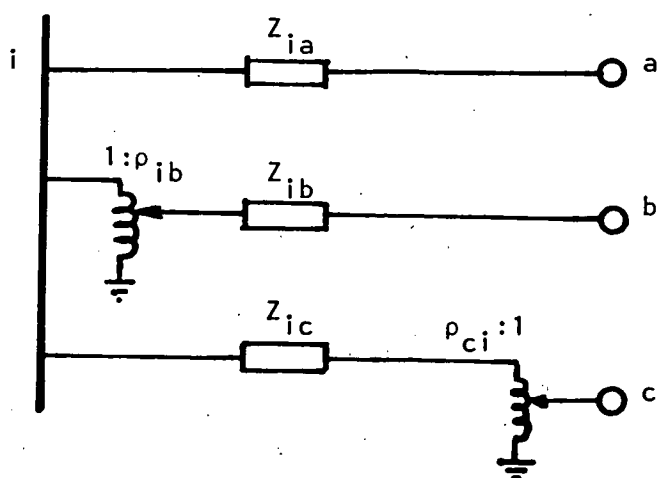


Fig 3.2 Three types of lines linking node i with adjacent nodes a , b , c

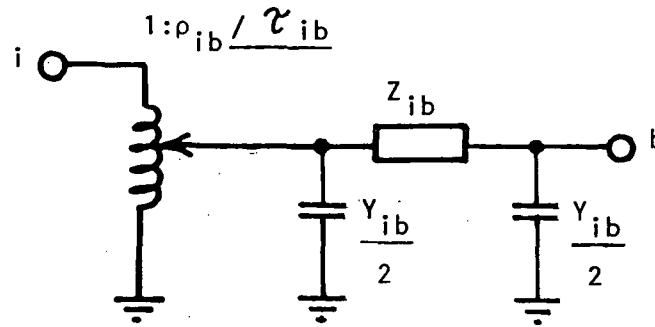
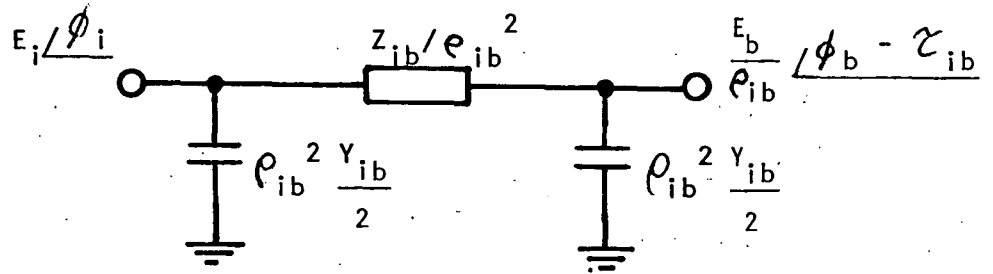


Fig 3.3 (a) Off-nominal turns transformer



(b) Equivalent - π model of off-nominal turns transformer as seen by bus i

$$Q_{ik} (E_{ik}, \phi_{ik}, \rho_{ijk}, \tau_{ijk}) = Q_{Gik} - Q_{Dik}$$

$$= - \sum_a \frac{E_{ik} E_{ak}}{Z_{ia}} \cos (\phi_{ik} - \phi_{ak} - \delta_{ia}) + \sum_a \frac{E_{ik}^2}{Z_{ia}} \cos \delta_{ia} - E_{ik}^2 \sum_a \frac{Y_{ia}}{2}$$

$$- \sum_b \rho_{ibk} \frac{E_{ik} E_{bk}}{Z_{ib}} \cos (\phi_{ik} - \phi_{bk} - \delta_{ib} + \tau_{ibk}) + \sum_b \rho_{ibk}^2 \frac{E_{ik}^2}{Z_{ib}} \cos \delta_{ib} - E_{ik}^2 \sum_b \rho_{ibk}^2 \frac{Y_{ib}}{2}$$

$$- \sum_c \rho_{ick} \frac{E_{ik} E_{ck}}{Z_{ic}} \cos (\phi_{ik} - \phi_{ck} - \delta_{ic} - \tau_{ick}) + \sum_c \frac{E_{ik}^2}{Z_{ic}} \cos \delta_{ic} - E_{ik}^2 \sum_c \frac{Y_{ic}}{2}$$

$$i = 1, 2, \dots, n$$

... (3.4)

or, in abbreviated form

$$P_{ik} (E_{ik}, \phi_{ik}, \rho_{ijk}, \tau_{ijk}) - P_{Gik} + P_{Dik} = 0$$

... (3.5)

$$Q_{ik} (E_{ik}, \phi_{ik}, \rho_{ijk}, \tau_{ijk}) - Q_{Gik} + Q_{Dik} = 0$$

... (3.6)

where ρ_{ijk} is the turns ratio of the transformer in line ij during time interval k

τ_{ijk} is the phase shift of the phase shifter in line ij during time interval k

3.3.2 Inequality constraints on the electrical subsystem

These are inequality constraints on power production P_{Gi} , reactive VA production Q_{Gi} , bus voltage magnitude E_i , voltage-phase-angle-differences $|\phi_i - \phi_a|$ between directly connected buses, total VA production S_{Gi} at each bus, transformer ratios ρ_{ia} , and phase shifter settings τ_{ia} .

Constraints on alternators: These are thermal limits on the steady-state stator current, usually expressed as an MVA constraint.

$$P_{Gi}^2 + Q_{Gi}^2 \leq \hat{S}_{Gi}^2 \quad \dots (3.7)$$

Turbine characteristics determine a minimum and maximum limit on generated power

$$\check{P}_{Gi} \leq P_{Gi} \leq \hat{P}_{Gi} \quad \dots (3.8)$$

Limits on reactive VA production are imposed, the maximum resulting from the thermal rating of the machine field, and the minimum resulting from stability considerations.

$$\check{Q}_{Gi} \leq Q_{Gi} \leq \hat{Q}_{Gi} \quad \dots (3.9)$$

Constraints on system voltage magnitudes: Large departures from nominal voltage in the system are undesirable and may cause tripping of some industrial loads. Voltage limits may also be imposed by statute or contract. An artificial constraint dictating maximum and minimum voltage magnitudes at each bus is therefore imposed.

$$\check{E}_i \leq E_i \leq \hat{E}_i \quad \dots (3.10)$$

A common voltage tolerance on industrial consumer busbars is $\pm 3\%$ but limits on most major system loads will be considerably wider, as distribution systems are usually supplied from tap-changing transformers.

Stability limits on lines: To ensure stability of the system, an upper limit is imposed on the phase angle between voltages at directly linked busbars.

$$|\phi_i - \phi_a| \leq \hat{\phi}_{ia} \quad \dots \quad (3.11)$$

Constraints on on-load tap-changing transformers: For the purpose of this study, the OLTC transformer is assumed to have ratio ρ_{ia} and phase shift τ_{ia} , both continuously variable between limits.

$$\check{\rho}_{ia} \leq \rho_{ia} \leq \hat{\rho}_{ia} \quad \dots \quad (3.12)$$

$$\check{\tau}_{ia} \leq \tau_{ia} \leq \hat{\tau}_{ia} \quad \dots \quad (3.13)$$

The usual OLTC transformer without phase-shifting capability is regarded as a special case with $\tau_{ia} = 0$.

3.3.3 Flow-balance constraints on the hydro-reservoirs

We can express the total discharge from a water storage j in time interval k as the sum of useful, bypass and spill discharges.

$$D_{jk} = D_{Gjk} + D_{Bjk} + D_{Sjk} \quad \dots \quad (3.14)$$

Spillway level imposes a maximum volume constraint on the reservoir.

$$V_{jk} = \min \{V_{ujk}, \hat{V}_j\} \quad \dots \quad (3.15)$$

V_{jk} is the volume of reservoir j at the end of time interval k and V_{ujk} is this volume if the upper bound \hat{V}_j is assumed infinite, so the spill can be expressed as

$$D_{Sjk} = (V_{ujk} - \hat{V}_j) \text{ step } (V_{ujk} - \hat{V}_j) \quad \dots \quad (3.16)$$

The flow balance equation for reservoir j over time interval k is

$$V_{ujk} = V_{j, k-1} + I_{jk} + \sum_{j_1} D_{j_1, k-\tau_{j_1}} - D_{Gjk} - D_{Bjk} - D_{Sjk} \quad \dots \quad (3.17)$$

where I_{jk} is the rivulet inflow of reservoir j in time interval k , $\{j_1\}$ is the set of reservoirs which are first upstream neighbours of reservoir j , and τ_j is the number of scheduling intervals elapsed before water released at reservoir j reaches its first downstream neighbour j_2 .

3.3.4 Inequality constraints acting on the hydro subsystem

For all hydro power plants, there are limits on the discharge rates and volumes of stored water. Useful discharge rate D_{Gjk} from reservoir j in time interval k is always in the following set of ranges. Range (iii) applies only to plants with pumping capability

$$\begin{aligned}
 \text{(i)} \quad & 0 \leq \check{D}_{Gjk} \leq D_{Gjk} \leq \hat{D}_{Gjk} \\
 \text{(ii)} \quad & D_{Gjk} = 0 \\
 \text{(iii)} \quad & \check{D}_{Gjk}^- \leq D_{Gjk} \leq \hat{D}_{Gjk}^- \leq 0 \quad \dots \quad (3.18)
 \end{aligned}$$

In general, these limits on discharge rates are different at different times of the year or even different time of day, and also depend on the amount of water present in the storages. This is because many reservoirs and tailwaters are part of irrigation schemes, are used for recreational purposes, or have statutory limitations imposed for scenery preservation. For cascade power plants, these restrictions also depend on the amount of water released at upstream and downstream plants. These may be, for the same reasons, limits on the bypass discharge of a storage,

$$\check{D}_{Bjk} \leq D_{Bjk} \leq \hat{D}_{Bjk} \quad \dots \quad (3.19)$$

Such limits are usually dependent on the time of day and season. As well as physical limitations on minimum useful reservoir volume, there may be irrigation, recreation or scenery preservation requirements imposed

$$\check{V}_j \leq V_{jk} \leq \hat{V}_j \quad \dots \quad (3.20)$$

These are not usually time-dependent in the short term.

3.3.5 Hydro-plant power vs discharge rate characteristics

These are covered in detail by 3.4 but may be expressed in the form

$$P_{Hik} = P_{Hi} (D_{Gik}, V_{Ajk}, V_{Aj_2k}) \quad \dots \quad (3.21)$$

for a hydro plant drawing water from storage j , discharging into reservoir j_2 and connected to bus i of the electrical network. V_{Aj_2k} is included to reflect tailwater elevation, and is zero if the associated flowtime $t_j > 0$.

P_{Hik} is the power generated by hydro means at bus i in time interval k ,

V_{Ajk} is the average volume of reservoir j in time interval k , and

j_2 is the reservoir immediately downstream of j , with flowtime delay $t_j = 0$. If $t_j \neq 0$, then j_2 is considered empty.

The head, h_{jk} , of reservoir j in time interval k either is constant or may be expressed as a polynomial function of storage volume.

$$h_{jk} = H_0 + H_1 V_{jk} + H_2 V_{jk}^2 + \dots + H_n V_{jk}^n \quad \dots \quad (3.22)$$

A linear or quadratic approximation is usually an adequate representation of the head vs storage volume relationship over its normal operating range.

3.3.6 Physical and imposed constraints

In consideration of inequality constraints on system state variables, Dillon [5, 6] drew a useful distinction between physical constraints which are inherent properties of the system such as maximum useful discharge of a hydro unit, and constraints imposed for operational reasons, often to prevent infringement of physical constraints such as insulation breakdown.

Violation of physical constraints is usually not possible without serious consequences, that is, they are "hard" constraints, while imposed constraints may on occasions be violated either of necessity as are voltage limits at the end of a long lightly loaded line, or because it is advantageous as in temporarily overloading a transformer, that is, imposed constraints are generally "soft" constraints.

"Hard" constraints on control variables are often easily implemented by holding the variable which has reached its limit at this limit while checking at each step of the optimisation that the variable has not left its limit, and forcing the change into adjoining variables [4, 11] or by variable interchange [7], for example transforming a P-V bus in a load flow calculation into a P-Q node when the limit of reactive generation is reached. Constraints in

other variables can be handled by transforming the problem formulation so that "hard" constraints become control variable constraints. For example:

Replace $x_i - u_j \leq 0$, a hard constraint and $u_j \leq u_j \leq \hat{u}_j$, a soft constraint by $u_j^1 \triangleq x_i - u_j \leq 0$, a hard constraint and $u_j \leq x_j - u_j^1 \leq \hat{u}_j$, a soft constraint.

"Soft" constraints can be implemented using penalty function methods [12], or by sequential unconstrained [8] or sequential linearly constrained [8] minimisation techniques. In operation of a practical power system, it is often useful to "protect" a physical constraint from violation by imposing a soft limit within the feasible region. By way of example although the maximum volume of a water storage is a physical limit, to treat it as such may result in considerable waste of energy due to spill. This may, in many cases be minimised by creating an artificial upper limit on storage volume, so allowing a little extra capacity for rainfall in excess of prediction, or operation a little off the requested setpoint.

3.4 REPRESENTATION OF HYDRO-ELECTRIC AND THERMAL POWER PLANTS

The input vs output curve establishes the relationship between the energy input to the driving system and the net energy output from the plant. For convenience, the input vs output relation is usually developed first on the basis of input vs alternator output, but for use in economic operation and interchange billing, this must be converted to input vs net plant output. In this conversion, only those auxiliary power requirements which are a function of whether or not the unit is operating and of the loading of the unit should be

charged against the unit for short-term scheduling purposes. Coal-handling, soot blowing and other intermittent operations are best handled by assigning at each load level an average requirement for sustained operation at that load. Methods used to establish input vs output curves are performance testing, determination from operating records and use of manufacturers guarantee data adjusted to actual operating conditions.

Performance testing for compliance with guarantee is in accordance with established codes. Periodic testing to obtain data for input vs output curves generally follows the same procedures. The main objection to such testing is its high cost, including testing labour, special instrumentation and the cost of replacing the lost output while the unit is being operated off-schedule. Determination from operating records is most practical for oil and gas fired thermal plant and hydro plant because of the relative ease of measuring the quantity and quality of fuel consumed, or the volume of water discharged. Use of manufacturers guarantee data assumes that the manufacturer provides a correct representation of the shape of the input vs output curve.

A performance correction factor is usually applied as a fixed percentage adjustment throughout the range of the curve, and is determined at regular intervals, usually weekly or monthly by comparing actual in-operation performance with the established curve after adjustment for boiler-banking, plant start-up and any change in quality of fuel [16]. The performance correction factor determined in this manner includes the effect of load following * as well as effects of turbine blade erosion and boiler fouling.

3.4.1 Representation of hydro-electric power plants

The performance characteristic of a hydro-electric power plant is a function of head losses in tunnels, pipelines or penstocks supplying the turbines, the tailwater elevation, the individual turbine characteristics and the distribution of load between turbines in the plant.

The common types of water turbine used for power generation are the Pelton, Francis, Kaplan and propeller wheels. Figures 3.4 and 3.7 are typical performance curves for these units [2, 9]. The Kaplan turbine, which has continuously adjustable runner blade angle, and the Pelton wheel, which is an impulse turbine, show superior performance characteristics to the fixed blade Francis turbine which develops cavitation at low discharge rates, and the propeller wheel which at part load suffers even more serious losses due to eddying and to velocity energy rejected. The Francis and propeller turbines are therefore only suited to operation at their maximum efficiency point, while Pelton and Kaplan turbines can be operated over a wide range of discharges with only a small reduction in efficiency.

3.4.1.1 Load division between units on line

In the operation of a hydro-electric power plant, the output corresponding to a given discharge rate depends on the division of

* Some turbo-alternators may have their governors set to hold system frequency constant, i.e., to follow the load, while others may be normally run with constant power output for extended periods, or have their power outputs manually controlled.

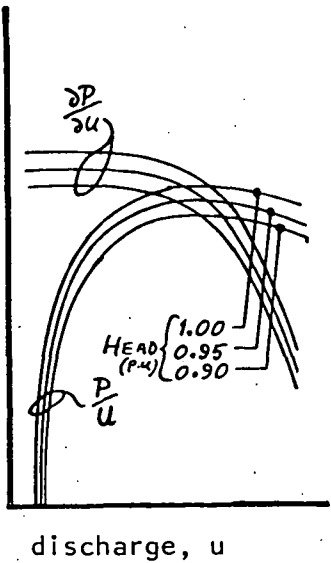
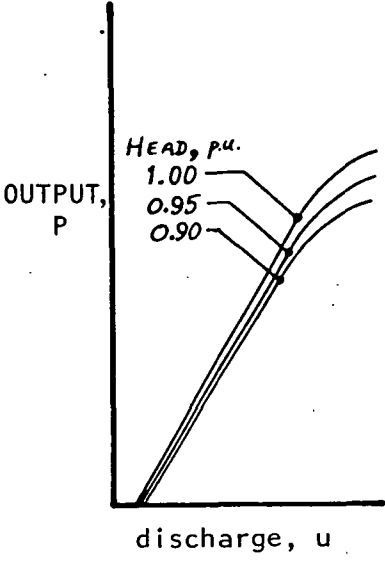
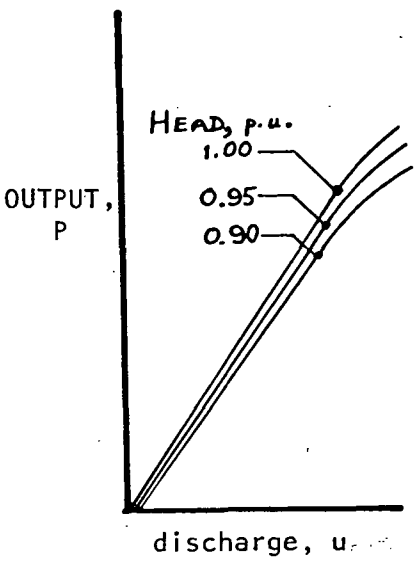


Fig 3.4 Pelton Wheel Characteristics

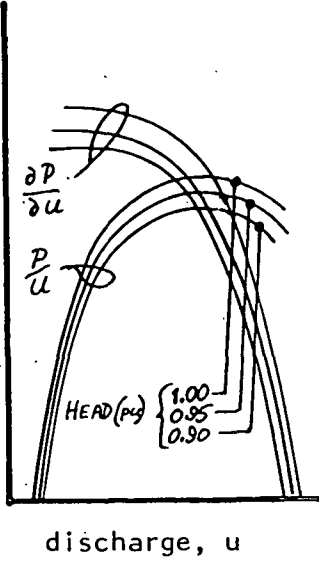


Fig 3.5 Francis Wheel Characteristics

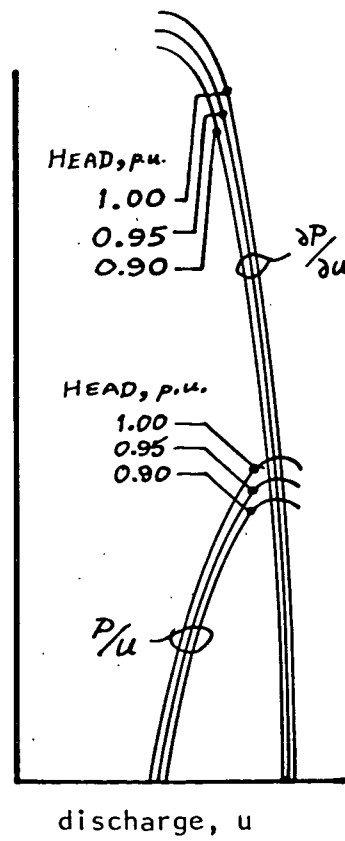
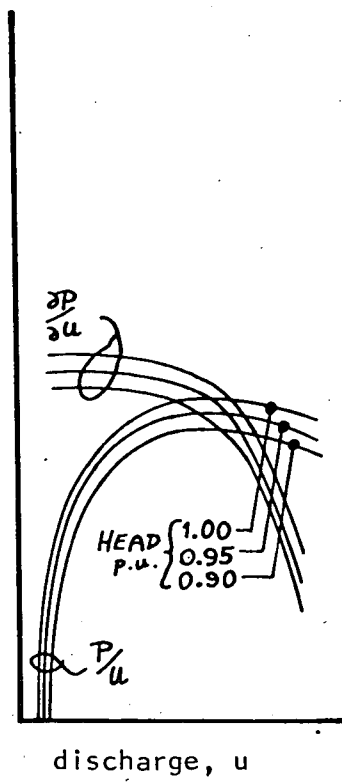
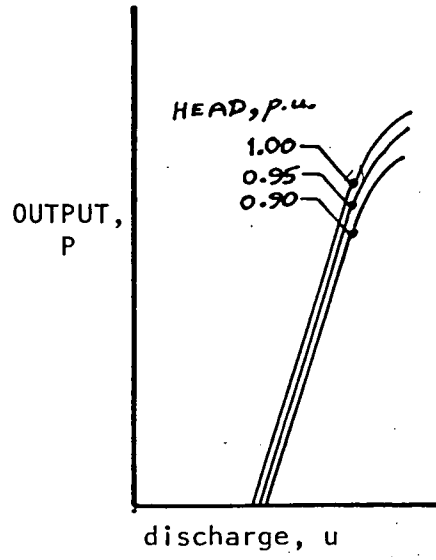
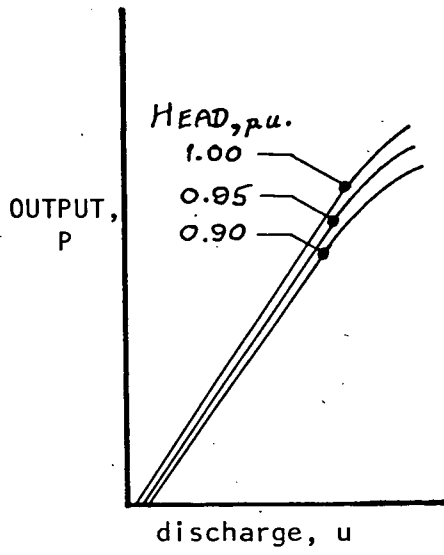


Fig 3.6 Kaplan Wheel Characteristics

Fig 3.7 Propeller Wheel Characteristics

discharge among the individual generating units. Units of a large hydro plant are usually identical, but may be divided into several (often two) groups of identical units. This often occurs where peaking machines have been added to a plant initially designed to supply base load. Optimal division of total discharge rate between the units of a hydro-electric plant is obtained by application of the principle of equal incremental specific power outputs to all units on line.

The head at the turbine is the plant static head minus tunnel, pipeline, flume and penstock losses. The plant static head is affected by the tailwater elevation. The specific output curve of the hydro plant must therefore reflect the variation of these losses with changes in total plant discharge rate and the individual unit discharge rates [1]. By way of illustration, consider a five-unit plant shown in Fig 3.8 with three units A having specific output curves of Fig 3.9(a) and two units B with specific output curves of Fig 3.9(b). These are supplied from the pond by a common tunnel, three common pipelines and separate penstocks with head-loss characteristics given in Figures 3.10 (a), (b) and (c) respectively. The units have a common tailwater with level rise characteristic of Fig 3.10(d). Combining all common head losses (tunnel, pipeline and tailwater level rise), we form a total common head loss curve, Fig 3.11, and combining penstock loss curves with unit specific output characteristics, the specific output curves for unit and penstock, Fig 3.12 (a) and (b) are formed. Finally, combining Figures 3.12 (a) and (b) whilst applying the principle of equal incremental rates, the plant specific output curve, Fig 3.13 is formed.

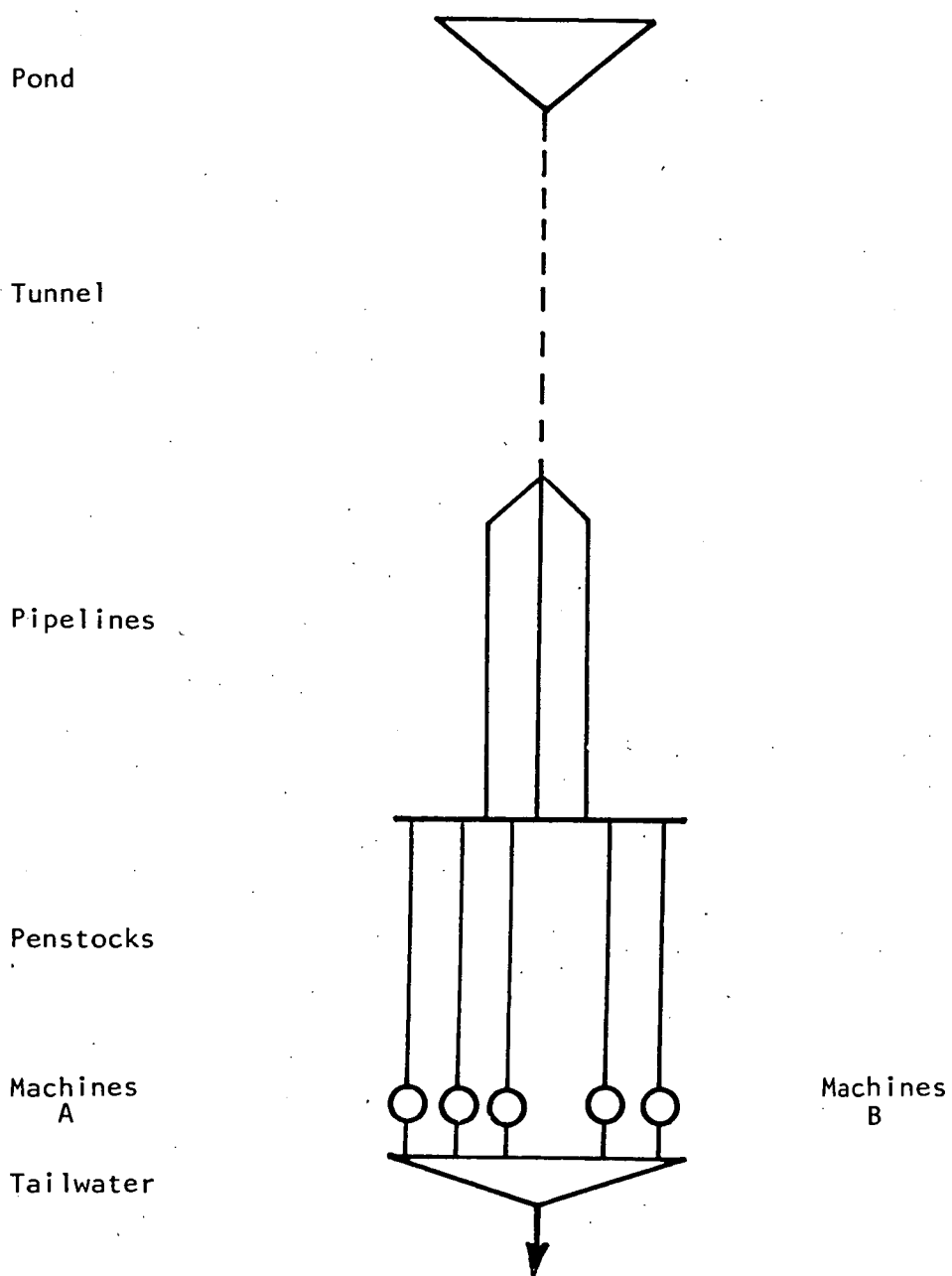


Fig. 3.8 Hydraulic diagram of a typical multiunit hydro plant

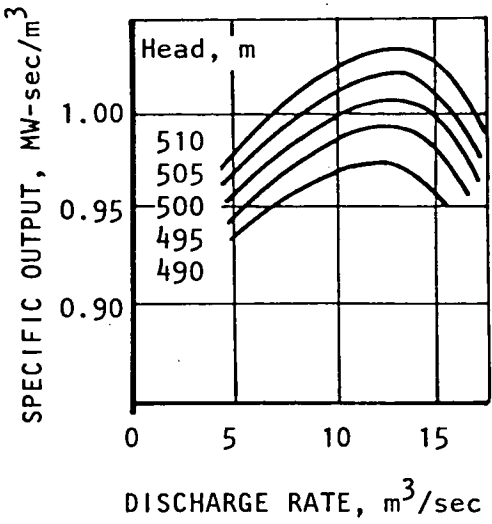
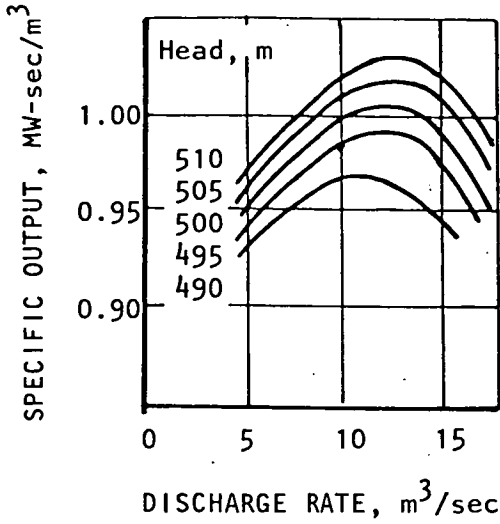


Fig 3.9 (a) Specific active generation of one unit type A



(b) Specific active generation of one unit type B

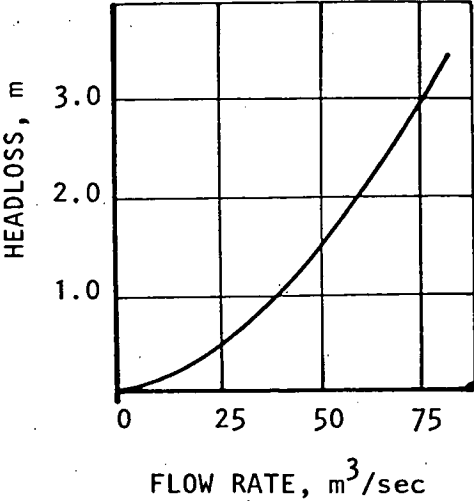
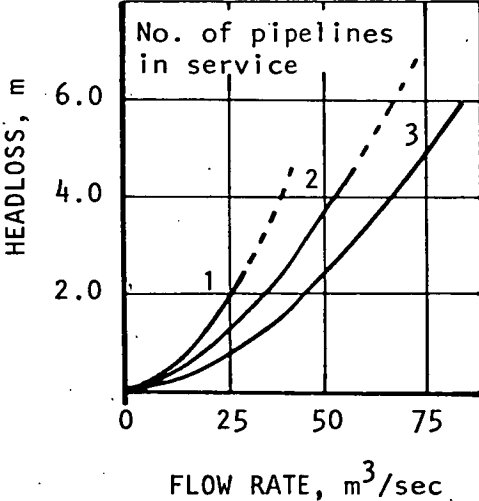
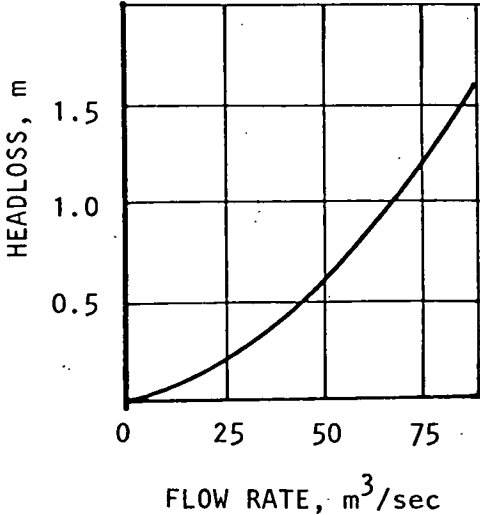


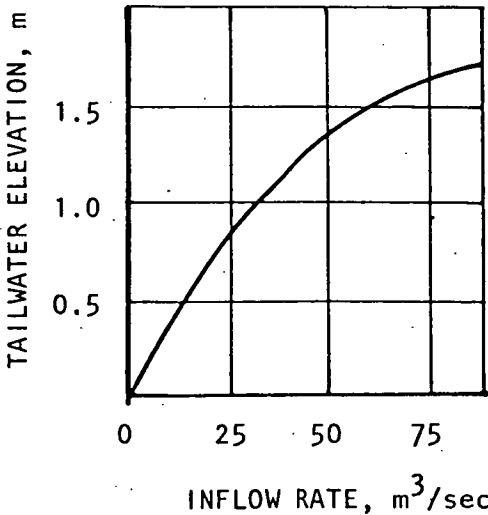
Fig 3.10 (a) Tunnel headloss



(b) Pipeline headloss



(c) Penstock headloss



(d) Tailwater elevation

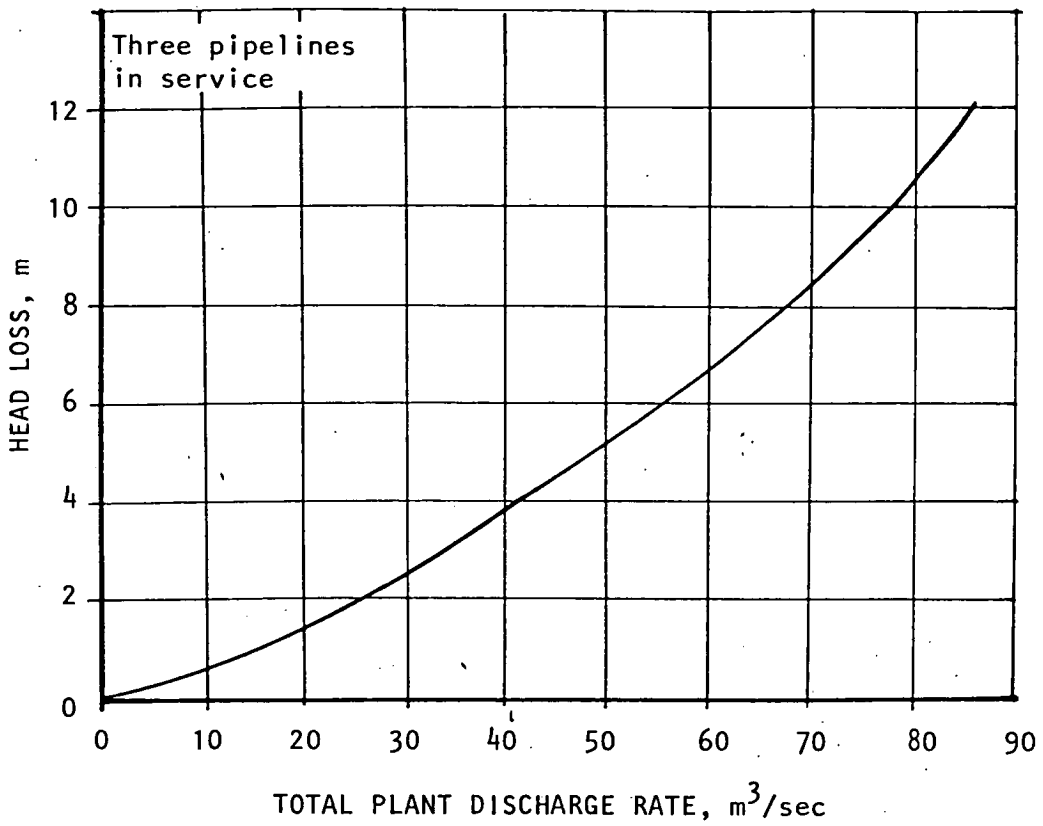


Fig 3.11 Total common headloss
(Tunnel + pipelines + tailwater)

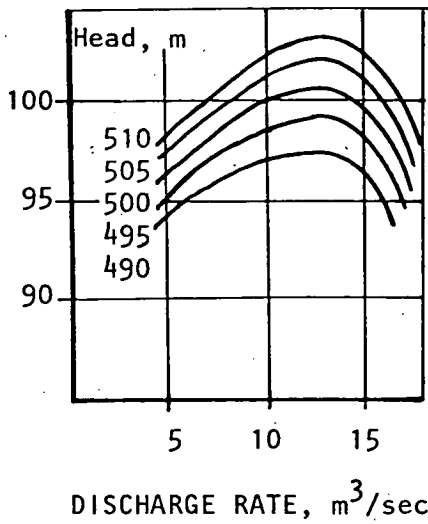
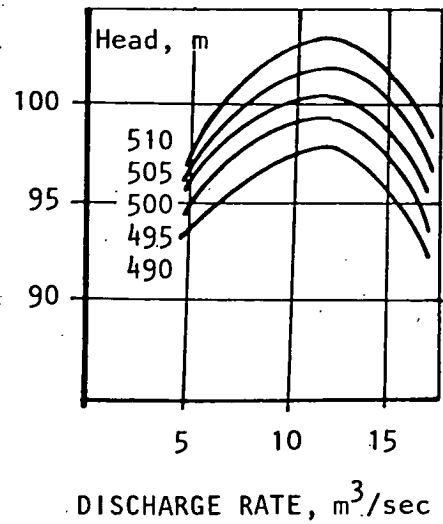


Fig 3.12 (a) Specific active generation of one unit type A and its penstock



(b) Specific active generation of one unit type B and its penstock

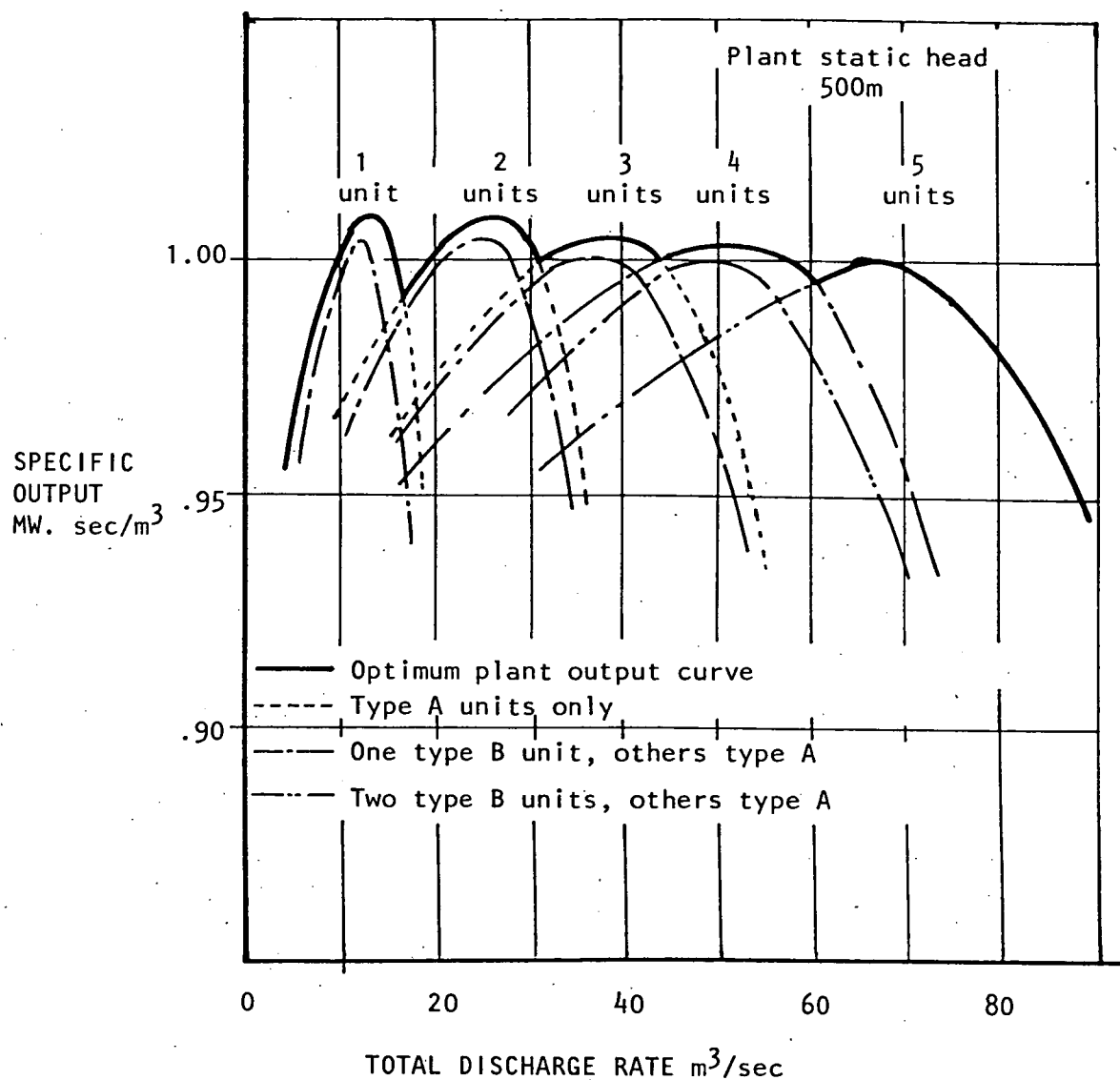


Fig 3.13 Composite optimised specific output curve of the 5-unit hydro plant

3.4.1.2 Variation of the number of turbines in service

Additional units must be brought into service as the load rises. The order of adding units, and the loads at which successive additions are made should be chosen so that the highest possible specific output (P/u) is maintained throughout the range of loads which the plant is capable of supplying.

Referring again to our example, suppose the plant has no units on line, and is required to assume a discharge rate of $12\text{m}^3/\text{s}$. Specific output curves for one unit of type A and for one unit of type B are calculated. The higher of the two curves indicates the unit to be brought into service first, in this case a type A unit. If the discharge is to be increased to, say, $25\text{m}^3/\text{s}$, then the specific output curves of each combination of two units (one of type A and one other) are calculated, assuming that the discharge is shared in accordance with the principle of equal incremental rates throughout the permissible range. Again, the higher specific output curve indicates the choice of unit to be brought into service. If the combined specific output curves for one unit and for two units intersect as in Fig 3.13, the point of intersection the load at which the second unit is added. If the curves do not intersect, the second unit is added when the first is fully loaded. The process is then repeated for addition of each extra unit. The resulting specific output curve of the plant and its derivation is illustrated by Fig 3.13.

Since the start-up and shut-down times of hydro-units are in general of the order of one to five minutes, and have little water use associated with them, it would usually not be economical to keep

a hydro-unit on line when not required either for supplying load or as spinning reserve, although some units are designed to function as synchronous reactors with zero water use. There is generally little or no reduction in maintenance ensuing from keeping a hydro-unit on line.

3.4.1.3 Mathematical description of a hydro power plant

Typical specific output curves for common types of hydraulic turbines are shown in Figures 3.4 and 3.7. These may be accurately approximated by a polynomial

$$P_{Gi}/U_{Gi} = a_0 + a_1 U_{Gi} + a_2 U_{Gi}^2 + \dots + a_n U_{Gi}^n \quad \dots (3.23)$$

Pipeline, tunnel and penstock head losses and tailwater elevation (Fig 3.10) are usually assumed to be quadratic functions of discharge rate [2], although the tailwater elevation curve is influenced greatly by the shape of the downstream channel. These are combined with the specific output curves of the turbines to form a specific output curve for the plant as in 3.4.1.2. A piecewise polynomial function of useful discharge can be fitted closely to this

$$P_{Gi}/U_{Gi} = a_0 + a_1 U_{Gi} + a_2 U_{Gi}^2 + \dots + a_n U_{Gi}^n; \check{U}_{Gi1} \leq U_{Gi} \leq \hat{U}_{Gi1}$$

$$P_{Gi}/U_{Gi} = b_0 + b_1 U_{Gi} + b_2 U_{Gi}^2 + \dots + b_n U_{Gi}^n; \hat{U}_{Gi1} \leq U_{Gi} \leq \hat{U}_{Gi2}$$

$$P_{Gi}/U_{Gi} = c_0 + c_1 U_{Gi} + c_2 U_{Gi}^2 + \dots + c_n U_{Gi}^n; \hat{U}_{Gi2} \leq U_{Gi} \leq \hat{U}_{Gi3}$$

etc.

... (3.24)

In almost all multi-unit hydro plants, the specific output curve is continuous, i.e., the $(i + 1)$ - unit is brought on line when the discharge rate corresponds to the intersection of the i -unit and $(i + 1)$ -unit specific output curves.

In high head plants, specific generation varies almost linearly with head, as turbine efficiency remains almost constant, but in low head plants, where head fluctuations can be a significant portion of the static head, a polynomial representation or interpolation between a set of models embracing the range of working heads may be desirable.

A hydro plant model representing the P_{Gi}/U_{Gi} vs P_{Gi} relation as a separate quadratic for each selection of units on line within that plant, and incorporating linear adjustment for total head is adequate for most hydro plants working with medium-to-high head.

As the economic life of a hydro plant is long and major rebuilds are infrequent, it is not necessary to make frequent adjustments to the plant model as with thermal plant. For accurate representation it is necessary, however, to make long term (often annual) allowance for the effects of penstock fouling, corrosion and erosion, and erosion and fouling of turbine blading, nozzles and casing. Plant deterioration also reduces the maximum power available from each unit.

3.4.2. Representation of thermal power plants [16]

Many recent thermal power plants are either large single-boiler single-turboalternator plants, or are built on a unit

principle, with one boiler for each turbogenerator without steam interconnection; while most older plants have relatively large numbers of small boilers and turbogenerators with steam interconnection.

3.4.2.1 Load division between boilers and turbines on line

When a plant is designed on the unit system, application of the principle of equal incremental rates to the combined boiler-turboalternator cost rate curve results in economic operation of that plant, but with full steam interconnection, the principle of equal incremental rates applied to the boiler and turbine halls independently results in overall economic operation. Most power plants have groups of identical boilers and turboalternators, and for these, the principle of equal incremental rates indicates that maximum economy is achieved if all running plant is loaded equally.

Boiler efficiency curves may be grouped into two main types [16] as in Fig 3.14 with incremental rate curves as in Fig 3.15. Figures 3.14 and 3.15 represent an ideal case, but in practice, performance and incremental rate curves are adjusted for the steam temperature characteristic of the boiler, auxiliary power consumption and steaming-rate dependent maintenance costs.

Similarly, there are two main types of turboalternator in current use. These are throttle governed turbines with or without overload valves, and partial admission nozzle governed turbines. Most steam turbines installed in Australia follow the British practice of throttle governing up to 80% of full load [16], and obtain the remaining 20% as overload capacity by bypassing the initial

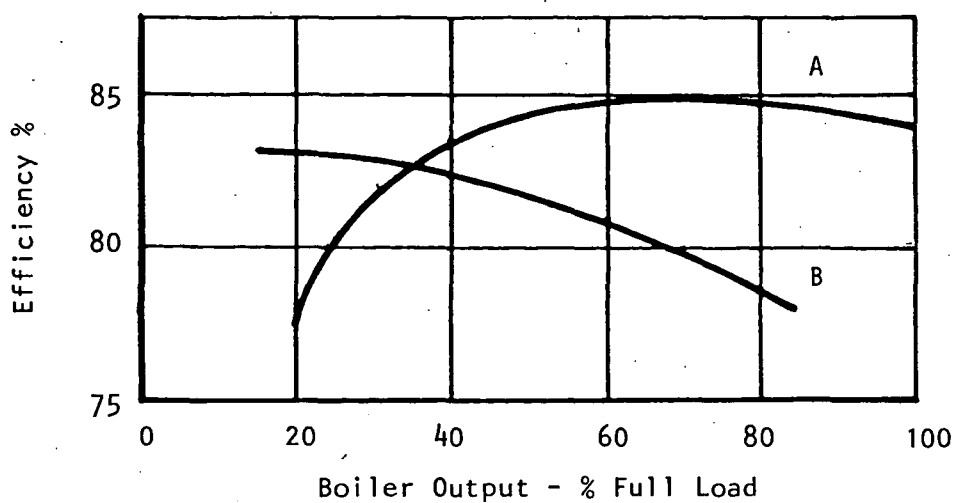


Fig 3.14 Efficiency curves for two boilers [16]

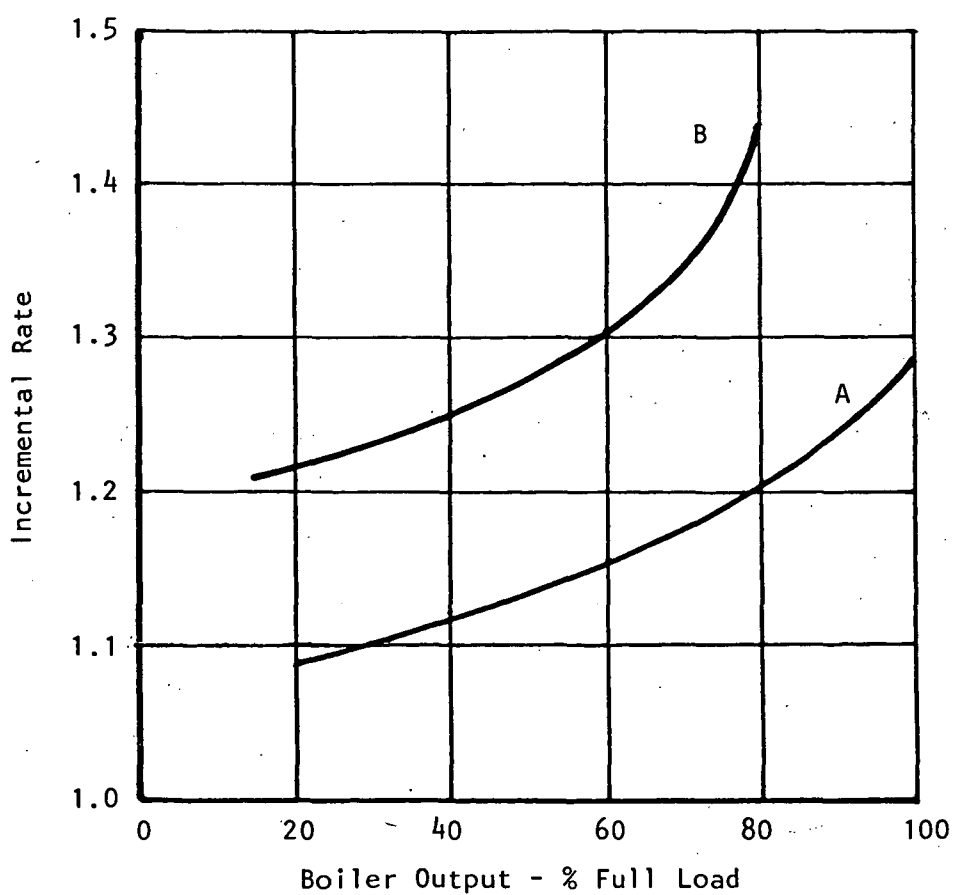


Fig 3.15 Incremental rate curves for two boilers [16]

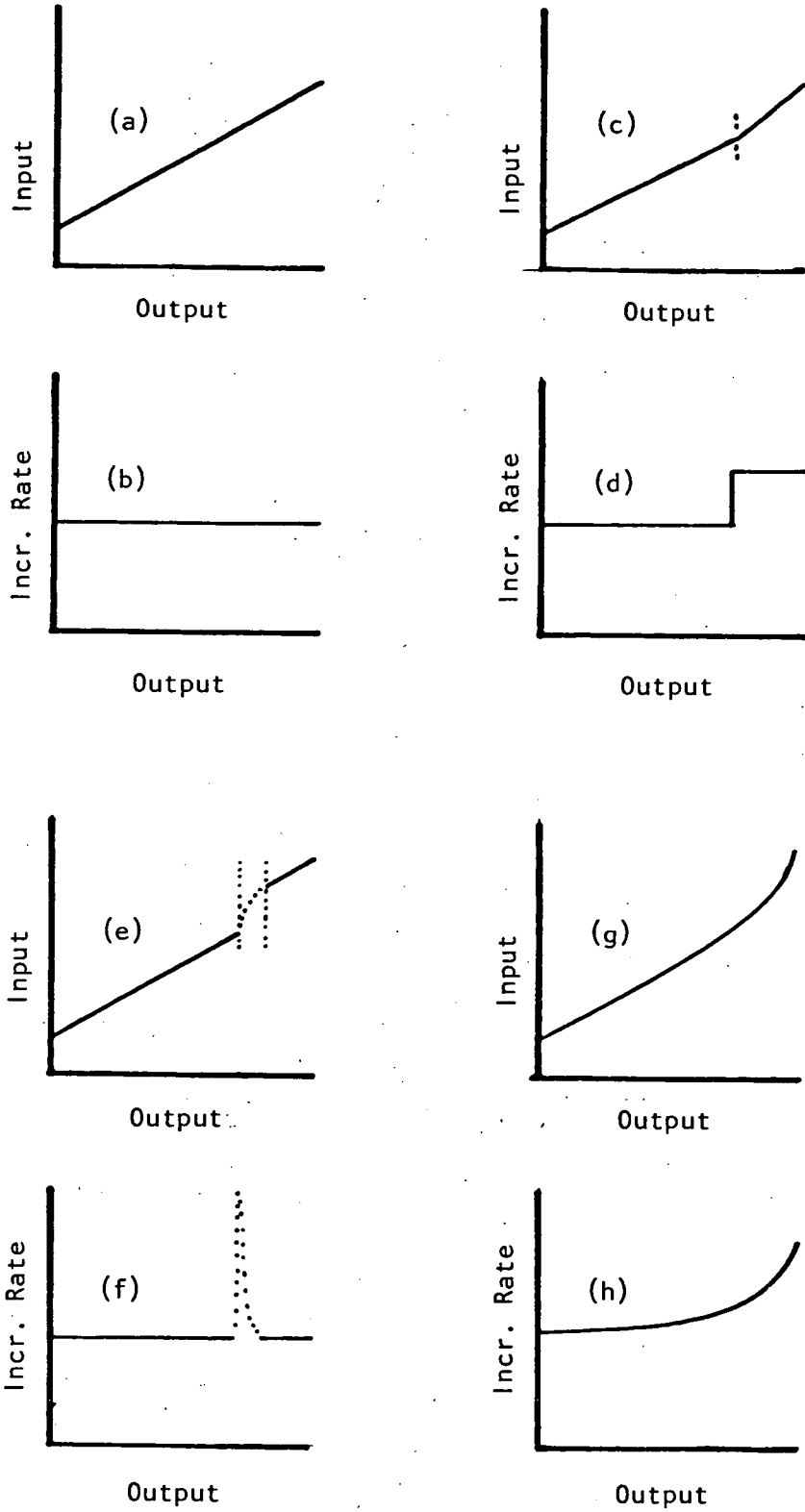


Fig 3.16 Some typical Input-Output and Incremental rate curves for steam turbines

stages with consequent lower efficiency. The performance curve (Willans line) of a throttle governed steam turbine essentially comprises one or more straight lines, depending on the number of governing valves, so the incremental rate curves are essentially horizontal line segments. Fig 3.16 shows several such performance and incremental rate curves. If overload valves are not operated, the combined efficiency of several units of this type in parallel is constant irrespective of load sharing. With partial-admission-nozzle governing [15], high efficiency is achieved at low output. Performance and incremental heat rate curves are similar to those of Figures 3.16(a) and (b) respectively. As with the boiler performance curves of Figures 3.14 and 3.15, the turbine characteristics of Fig 3.16 are for ideal uses. Corrections must be made for variations in circulating water temperature and auxiliary power consumption.

3.4.2.2 Variation of the number of boilers and turbines in service

In a plant designed on the unit system, selection of boiler-turboalternator units and the loads at which they should be brought on line may be determined by a similar process to that used in 3.4.1.2 for selecting and loading hydro-units, while in a plant with full steam interconnection, the same procedure is applied separately to boiler and turbine hall loading. This is illustrated in Figures 3.17 and 3.18 with reference to a turbine room containing four similar 50MW turboalternators fitted with first stage bypass valves. A fall in load may require withdrawal of plant from service. The optimum order is the reverse of bringing it into service.

When deciding to take a boiler out of service, a comparison

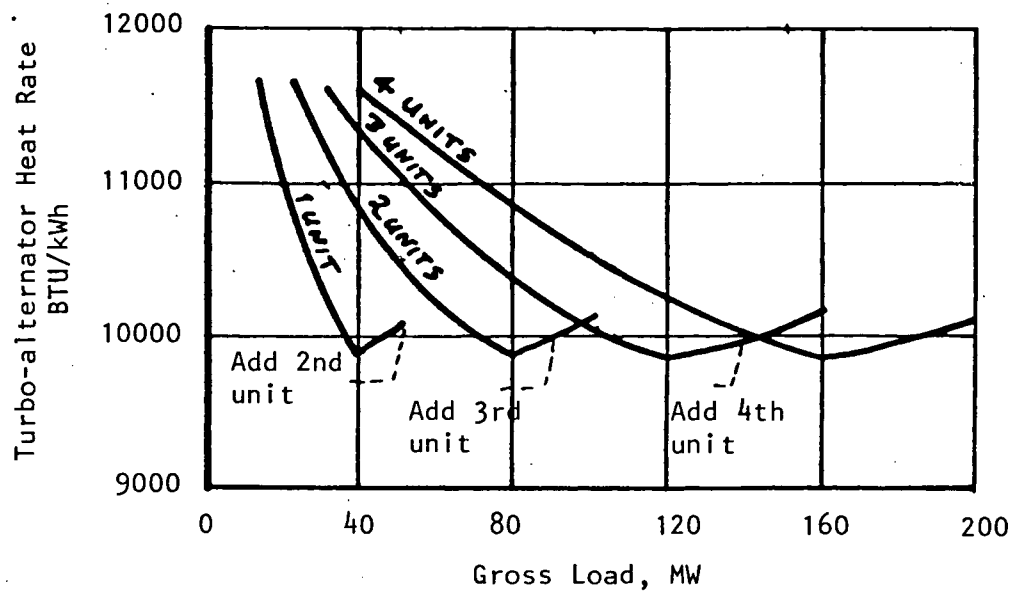


Fig 3.17 Heat rate curves for a turbine room of four similar 50MW turboalternators fitted with bypass valves [16]

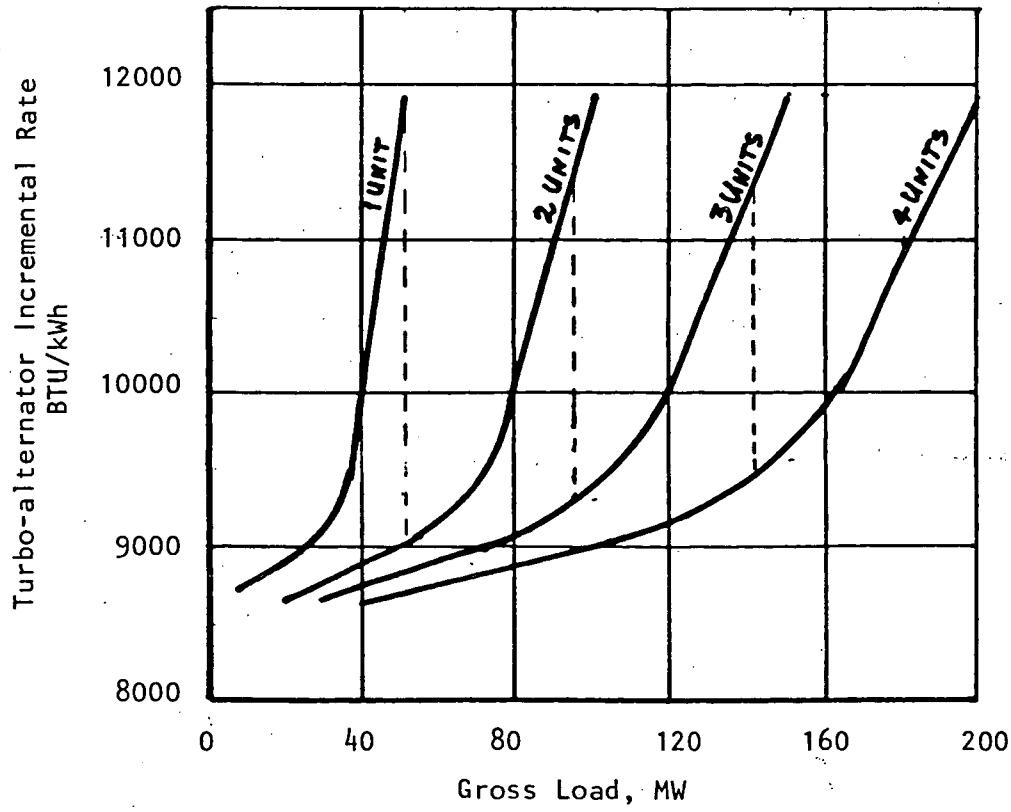


Fig 3.18 Incremental rate curves corresponding to Fig 3.17 [16]

must be made between fact savings by reducing the number of steaming boilers and the amount of fuel required to restore the plant to steaming condition. If boilers have performance curves similar to that of boiler B in Fig 3.14, the operating efficiency is improved by reducing load and the minimum steaming rate, rather than operating economy, determines when boilers are taken out of service. In a similar way, the increase in efficiency obtained by removing a turboalternator from line must be balanced against the cost of running up and resynchronising. Starting and stopping plant increases the liability to breakdown primarily due to thermal and pressure cycling, so increasing maintenance costs. Consequently, even if a fuel economy study shows a small margin in favour of taking plant out of service, it would frequently pay to keep it running to improve reliability and reduce maintenance costs [16].

3.4.2.3 Mathematical description of a thermal power plant

The production cost vs power output curves both for a single generating unit and for a thermal power plant may be approximated by a piecewise polynomial function.

Thus

$$F(P_{Gi}) = a_0 + a_1 P_{Gi} + a_2 P_{Gi}^2 + \dots + a_n P_{Gi}^n; \check{P}_{Gi1} \leq P_{Gi} \leq \hat{P}_{Gi1}$$

$$F(P_{Gi}) = b_0 + b_1 P_{Gi} + b_2 P_{Gi}^2 + \dots + b_n P_{Gi}^n; \hat{P}_{Gi1} \leq P_{Gi} \leq \hat{P}_{Gi2}$$

$$F(P_{Gi}) = c_0 + c_1 P_{Gi} + c_2 P_{Gi}^2 + \dots + c_n P_{Gi}^n; \hat{P}_{Gi2} \leq P_{Gi} \leq \hat{P}_{Gi3}$$

etc. ... (3.25)

In modelling a single generating unit, it will often suffice to use a single polynomial over the whole range of operation, a partial-admission-nozzle-governed plant being accurately represented by a single quadratic [14]. For throttle-governed plant, a discontinuous piecewise linear approximation is commonly used [14] (see Fig 3.16).

It is necessary to make periodic adjustments to the model to compensate for erosion of turbine blades, fouling of the boiler and other deterioration. These adjustments are made on the basis of performance records and usually take the form of a "performance factor" applied to the production cost. Plant deterioration also reduces the maximum power output available from each unit. Re-calculation of the performance factor of thermal power plant is typically at monthly intervals.

3.5 RECAPITULATION

This chapter has formulated a mathematical model of a power system which is complete in those details relevant to short-term scheduling. It is framed for optimal control of an integrated power system, but is adaptable to hydro or thermal systems by deleting reference to the type of plant not present. Provision for purchased power is included. Optimality criteria appropriate to short-term scheduling of both hydrothermal and hydro systems were discussed, primarily with reference to total water use at each hydro plant (except at the slack bus plant in a predominantly hydro system) being predetermined by the medium and long term scheduling phase,

Criteria for the optimal operation of units within plants were developed, allowing simplification of hydro plant characteristics from individual machine characteristics, head drop characteristics and the like to a single set of curves for each plant with an adjustment formula for changes in total head. Similarly, thermal plant characteristics are simplified from individual boiler and turboalternator characteristics to a single set of performance curves for the plant. The representation of each hydro plant need not be changed frequently as performance deterioration is gradual, but with thermal plant, a performance factor, often adjusted monthly, is applied. Allowance for variation in fuel quality and cooling water temperature must be made.

The equality constraints of load balance in the power transmission system and water-flow balance in the river system are included together with all inequality constraints on power generation, transmission and river system components. Contract and statutory limitation are also included. The distinction drawn by Dillon [5] between physical and imposed constraints on the system is discussed together with the frequent desire to impose constraints inside actual physical constraints to allow a margin of safety which can in exceptional circumstances be violated by a small amount.

The inclusion of all necessary constraints and use of full representation of generating plant ensures that feasible schedules will always result when the complete model is used.

This model is far too complex for the unit commitment process, and simplifications as detailed in 4.2 have been made so that the step-loading scheme for hydro unit-commitment and pre-

scheduling described in Chapter 4 can be economically implemented.
The complete model is retained for the actual scheduling process.

3.6 REFERENCES

- [1] Adams, R.L., Economic load flow control, Residential School in power system electrical engineering, University of Tasmania, 1975.
- [2] Addison, H., A treatise on applied hydraulics. Chapman and Hall, London, 1956.
- [3] Anstine, L.T. and Ringlee, R.V., Susquehanna river hydro-thermal co-ordination. IEEE Trans. Power Appar. Sys. Vol. 82, No. 65, April, 1963, pp 185-191.
- [4] Carpentier, J. and Sirieux, J., L'optimisation de la production a Electricite de France. Bull. de la Soc. Fr. des Electriciens, V. 4, 1963, pp 121-129.
- [5] Dillon, T.S., Problems of optimal economic operation and control of integrated (hydrothermal) and thermal power systems. Ph.D. thesis, Electrical Engineering Department, Monash University, April 1974.
- [6] Dillon, T.S. and Morsztyn, K., A new theoretical and computational approach to the exact solution of the problem of optimal control of integrated (hydro-thermal) power systems. 5th IFAC World Congress, Paris, June 1972.

- [7] Dommel, H.W. and Tinney, W.F., Optimal power flow solutions. IEEE Trans. Power Appar. Sys. Vol. 87, No. 10,, Oct. 1968, pp 1866-1876.
- [8] Fiacco, A.V. and McCormick, G.P., Nonlinear Programming: Sequential unconstrained minimisation techniques. John Wiley and Sons, New York, 1968.
- [9] IEEE Committee Report, Present practices in the economic operation of power systems. IEEE Trans. Power Appar. Sys., V. 90 No. 2, 1071, pp 1768-1775.
- [10] Kirchmayer, L.K., Economic control of interconnected systems John Wiley and Sons, 1959.
- [11] Peschon, J., Piercy, D.S., Tinney, W.F., Tsviet, O.J. and Cuenod, M., Optimum control of reactive power flow, IEEE Trans. Power Appar. Sys. V. 87 No. 1, 1968, pp 40-48.
- [12] Pierre, D.A., Optimisation theory with applications. John Wiley and Sons, 1969.
- [13] Sakkappa, B.G., Optimal hydro-thermal scheduling with a pseudo-thermal resource. IEEE Proc. PICA., 1963, pp 354-360.

- [14] Steinberg, M.J. and Smith, T.H., Economy Loading of power plants and electric systems. John Wiley and Sons, New York 1943.

- [15] Stott, B., Humpage, W.D. and Braemeller, A. Optimal load despatching solutions. Symposium on optimal power system operation, University of Manchester, 1969.

- [16] Tefler, J.M., Integrated system operation - Economy. Symposium on power system operation, University of Sydney, 1967.

4. A STEP-LOADING TECHNIQUE FOR UNIT COMMITMENT IN THE HYDRO SUBSYSTEM

For the hydrothermal optimisation techniques of Chapter 5 to be effective, the cost curves must be locally convex. As the generation vs discharge rate characteristics of multi-unit hydro plants have discontinuous slopes, hydro unit commitment must be established for each time interval to ensure convexity. A suitable and commonly used technique [1, 8] producing an initial schedule for more complex methods to commence from and establishing unit commitment is step-loading, restricting the discharge rate of an m -unit plant to $m + 2$ levels usually corresponding to zero discharge, efficiency peaks, and full gate discharge.

In an ideal system where

- (i) instantaneous transmission losses vary negligibly with generation pattern
- (ii) thermal generation cost characteristics are linear in a hydrothermal system, or the slack-bus hydro plant characteristics are linear in an all-hydro system
- (iii) each reservoir has negligible head variation, and for daily allowable reservoir depletion specified in advance by a longer term schedule, optimal operation of each plant is step-loading between two discharge rates. This is justified heuristically in 4.1. Mantera [7, 8] attempted a justification based on Pontryagin's maximum principle, but this was invalidated by a mathematical error. The error is rectified in appendix 1 of this thesis.

4.1 STEP-LOADING IN AN IDEAL HYDRO SUBSYSTEM

Consider a multi-plant hydro subsystem conforming to the preceding ideal conditions. The plant characteristics are

$$P_{Gi} = \psi_i(u_i); i = 1, 2, \dots, n_H - 1 \quad \dots (4.1)$$

$$u_{n_H} = a P_{Gn_H} + b; a \text{ \& } b \text{ constants} \quad \dots (4.2)$$

The total generation constraint is

$$P_{Gn_H} = P_D + P_{TL} - \sum_{i=1}^{n_H-1} P_{Gi} \quad \dots (4.3)$$

where P_{Gi} is the power output of plant i ,

n_H is the number of hydro plants in the subsystem,

u_i is the discharge rate of hydro plant i ,

P_D is the system active load,

P_{TL} is the active transmission loss (independent of generation pattern).

As system active load is not usually alterable and transmission loss is independent of generation pattern, total electrical energy demand is fixed at any time. Therefore, by maximising the efficiency of the base-load plants ($i = 1, 2, \dots, n_H - 1$), minimum electrical energy is drawn from the slack-bus plant ($i = n_H$). Since its power vs discharge rate characteristic is linear, water drawn from the frequency control plant storage is a minimum. As shown below, step-loading gives maximum-efficiency operation of an isolated plant, hence yielding an optimal schedule for this ideal system.

It is necessary to select switching times of the base-load

plants so that the spinning reserve requirements are not infringed.

4.1.1 Maximum Efficiency Operation of a Single Unit Hydro Plant^[6]

Consider a single unit hydro plant attached to an infinite bus, with the constant-head generation vs discharge-rate characteristic of Fig. 4.1. Any schedule must comprise operation on this curve, or of switching between points on it. If the schedule comprises switching between two points on the characteristic curve, the average generation vs average discharge rate characteristic is represented by the chord joining the two points.

If $0a$ is the tangent to the characteristic at $a(u_a, P_a)$, over the range $[0, u_a]$, the generation vs discharge rate curve lies entirely below chord $0a$, as does any chord representing step-loading between discharge rates other than 0 and u_a . Over the range $[u_a, u_{\max}]$, any chord joining two points on the generation vs discharge rate curve lies below that curve. The most efficient mode of independent operation of a single unit plant is, for $0 \leq u_{av} < u_a$, to step-load between discharge rates $u = 0$ and $u = u_a$, where u_a is the peak efficiency discharge rate, and for $u_a \leq u_{av} \leq u_{\max}$, continuous operation at discharge rate $u = u_{av}$. The resulting average generation vs average discharge rate characteristic is the broken line $0am$ in Fig. 4.1.

4.1.2 Maximum Efficiency Operation of a Multi-Unit Hydro Plant^[6]

Consider a two-unit hydro plant attached to an infinite bus, with the constant head generation vs discharge rate characteristic of Fig. 4.2. $0a$ is tangent from $(0, 0)$ to the single-unit curve and bc

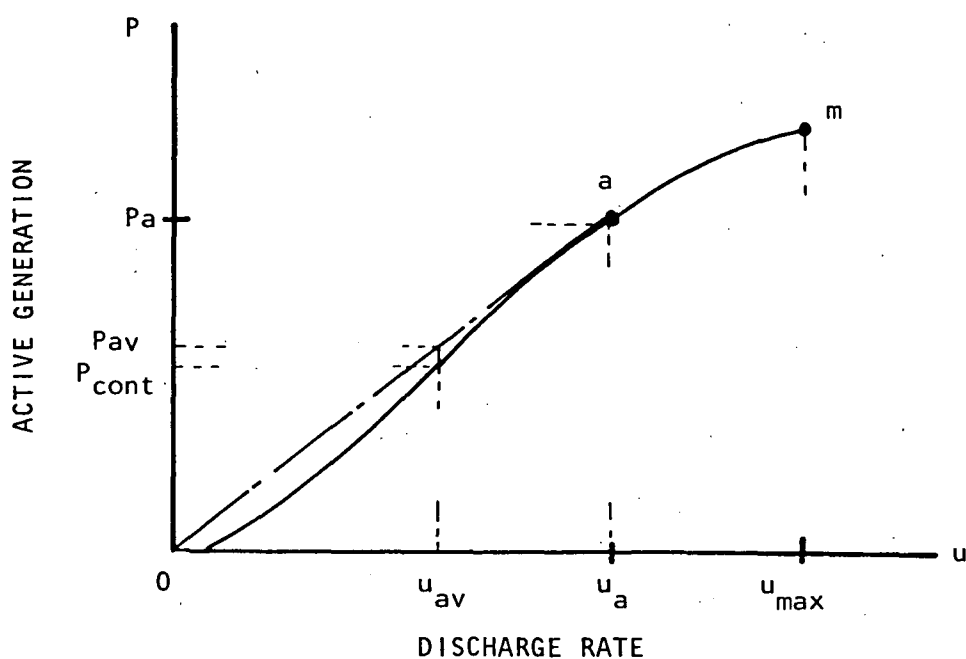


Fig 4.1 Active generation vs discharge rate characteristic of a typical single unit plant operating at fixed head (exaggerated nonlinearity)

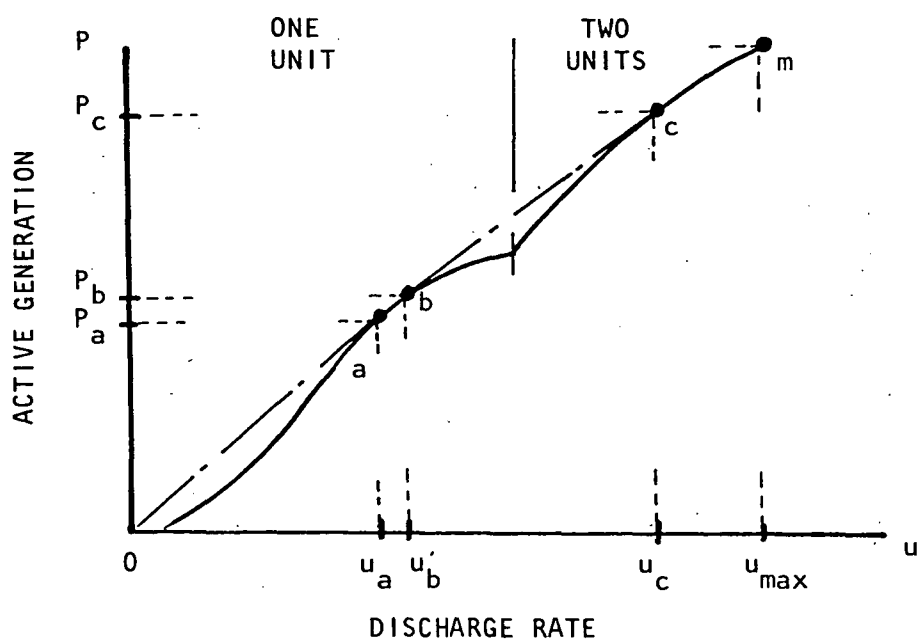


Fig 4.2 Active generation vs discharge rate characteristic of a typical two unit plant operating at fixed head (exaggerated nonlinearity)

is the common tangent to the one and two unit curves. Chords 0a and bc correspond to average generation vs average discharge rate characteristic for step-loading between $(0, 0)$ and (u_a, P_a) and between (u_b, P_b) and (u_c, P_c) respectively.

By analogy with the single-unit plant, most efficient generation within the ranges $0 \leq u_{av} \leq u_a$ and $u_b \leq u_{av} \leq u_c$ is obtained using the step-loading modes proposed above, while if $u_{av} \geq u_c$, continuous operation at (u_{av}, P_{av}) is indicated. Within the range $u_a \leq u_{av} \leq u_b$, any chord connecting two points on the generation vs discharge rate curve lies below curve ab so it is most efficient to operate continuously at (u_{av}, P_{av}) .

It is simple to extend this reasoning to a general multi-unit plant.

4.2 APPLICATION OF STEP-LOADING TO NON-IDEAL SYSTEMS WITH VARIABLE TRANSMISSION LOSS

Although analysis indicates that the overall efficiency of a hydro plant operated in the step mode is independent of the number of switchings of discharge rate, practical considerations limit their frequency. Other important factors in a real system are dependence of transmission losses on generation pattern and the slack bus plant characteristics which are nonlinear and, for multi-unit plants, have discontinuous slope.

To apply step-loading effectively to non-ideal hydro-thermal systems, the thermal plants must have smooth cost characteristics. The times at which base-load hydro plants change their

output level are then chosen to minimise total thermal plant fuel cost whilst satisfying water use constraints on hydro plants and leaving adequate spinning reserve in thermal plants to cover forecast errors.

In a totally hydro power system, the slack-bus hydro plant must have a reservoir large enough that head variations need not be considered*. The times at which base load hydro plants change their output level are chosen to minimise total water use by the slack bus plant whilst satisfying water use constraints on the base load plants and leaving adequate spinning reserve in the slack bus plant to cover forecast errors.

Step-loading as a means to a good suboptimal schedule on which to base a search for the optimal schedule does not justify a precise representation of transmission losses, as it involves far more computing than if a well chosen set of B-coefficients [9] were used.

Where the slack-bus plant is a multi-unit hydro plant with the characteristics of Fig. 4.3 Curve A, it should be modelled by a smooth curve as in Fig. 4.3B, eliminating spurious minima. This is justified since we are obtaining a starting schedule for the final optimisation process, but will not at this stage know the final output of the slack bus plant accurately enough to determine with certainty the number of slack bus plant units loaded.

* [8] In all-hydro power systems, the slack-bus plant is frequently a high head multi-unit plant with large storage, and has a characteristic similar to Fig. 4.3.

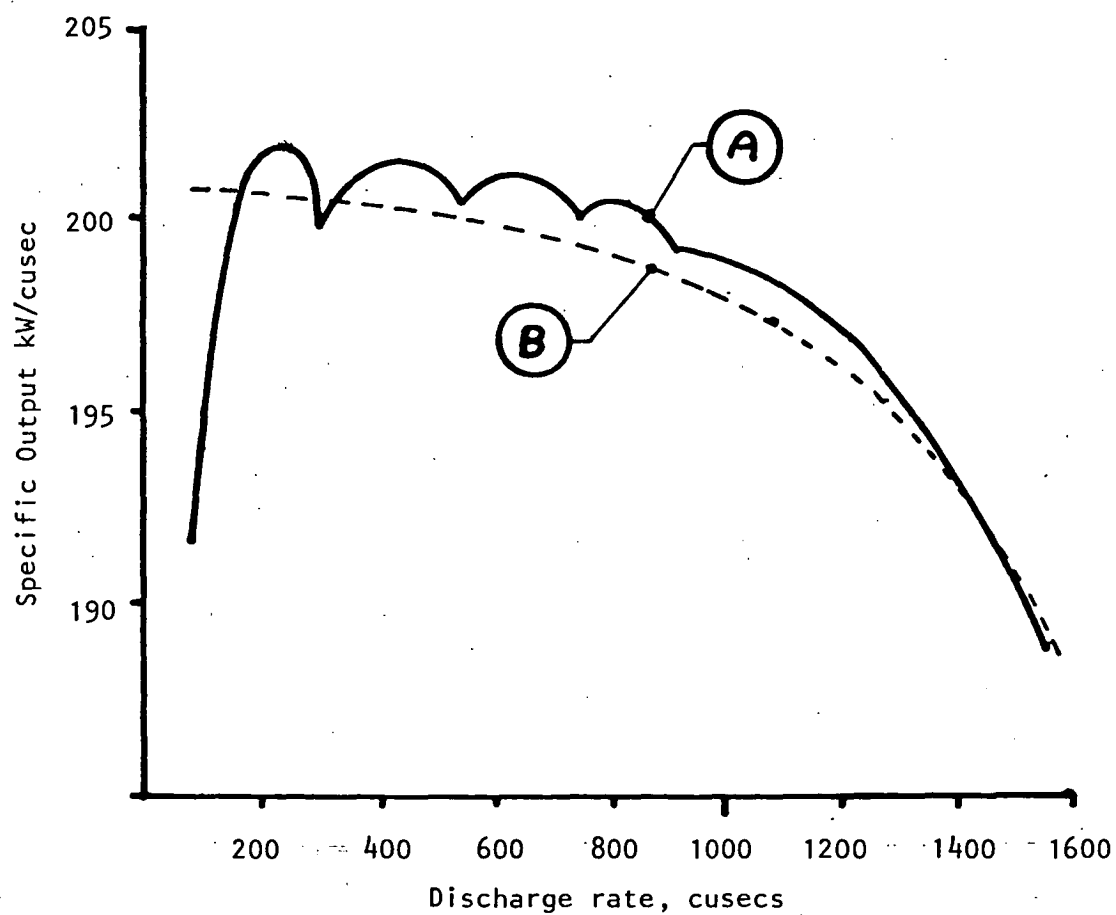


Fig 4.3 Approximation of a multi-unit slack bus hydro plant characteristic by a smooth curve

4.3 COMPUTATIONAL ASPECTS OF STEP LOADING

Consider a power system of four base loaded hydro plants and a slack bus plant, either hydro or thermal, scheduled at hourly intervals over 24 hours. There are up to C_{12}^{24} or 2.7_{10}^6 possible schedules for each step loaded base generation plant and therefore up to $(C_{12}^{24})^4$ or 5.3_{10}^{25} step loading schedules for a five plant system.

The step-loading problem is multi-minimal as noted in 4.3.2 so local minima-seeking search methods may lead to non-optimal schedules. We are therefore restricted to a global search or those searches which fully examine the subset of schedules in which the optimal schedule lies. The global search must be discarded because of the dimension involved.

Mantera [7, 8] restricted each plant to two changes of discharge rate within the time span of scheduling which, for the five plant system discussed above, would lead to $24^4 = 3.3_{10}^5$ schedules being examined. This is well within the scope of evaluation, but does not yield the optimal schedule in many cases.

Consider a system one stage removed from an ideal system, with transmission losses independent of generation pattern and the slack bus incremental cost rate related to the square of output. The base load plants are, as for the ideal system, scheduled to step between two discharge rates such that the total discharge of each plant complies with a longer term schedule. The total cost (or discharge) of the slack bus plant over the scheduling period,

$$\$_{ToT} = \int_0^T \frac{d\$_t}{dP_t} \cdot dt = \int_0^T (k_1 + k_2 P_t + k_3 P_t^2) \cdot dt \quad \dots (4.4)$$

where $\$_t$ = cost (or water use) rate of the slack bus plant at time t within the scheduling period T

P_t = output power of the slack bus plant at time t within the scheduling period T

k_1 , k_2 and k_3 are constants.

Defining ΔP_t by

$$P_t = P_{av} + \Delta P_t \quad \dots (4.5)$$

$$\text{where } P_{av} = \frac{1}{T} \int_0^T P_t \cdot dt \quad \dots (4.6)$$

and P_{av} is independent of schedule since the system transmission losses are constant, (4.4) becomes

$$\begin{aligned} \$_{ToT} &= \int_0^T (k_1 + k_2 P_{av} + k_2 \Delta P_t + k_3 P_{av}^2 + 2k_3 P_{av} \Delta P_t + k_3 \Delta P_t^2) \cdot dt \\ &= T (k_1 + k_2 P_{av} + k_3 P_{av}^2) + (k_2 + 2k_3 P_{av}) \cdot \int_0^T \Delta P_t \cdot dt \\ &\quad + k_3 \int_0^T \Delta P_t^2 \cdot dt \quad \dots (4.7) \end{aligned}$$

$$\int_0^T \Delta P_t \cdot dt = 0 \text{ and putting } k_4 = T (k_1 + k_2 P_{av} + k_3 P_{av}^2)$$

$$\$_{ToT} = k_4 + k_3 \int_0^T \Delta P_t^2 \cdot dt \quad \dots (4.8)$$

In order to minimise total cost (or total discharge) of the slack bus plant, we must minimise $\int_0^T \Delta P_t^2 \cdot dt$. This integral is zero if the slack bus plant operates at constant load. In reality, to minimise cost, we need to match the shape of the system load curve

with the base load plants. This can be achieved by applying the restrictions of two changes of discharge rate within the scheduling period to the system load vs duration curve rather than to the chronological load curve.

Dillon ^[2] also noted the tendency for long-term schedules to yield an almost constant slack bus plant output.

4.3.1 Five Plant Hydrothermal and Hydro System Examples

The five plant system of Fig 4.4 was studied. System transmission losses were represented by George's simplified loss formula ^[5] with B-coefficients tabulated in Fig 4.5.

Each of the four base-loaded hydro plants are operated at their individual optimum by step-loading between two discharge rates subject to a total discharge constraint as described in Section 4.1. Fig 4.6 tabulates the step-modes and total discharge constraints of the four base load plants. Two different slack bus plants are considered: a single unit thermal plant with a cost characteristic given by Fig 4.7 and a five-unit hydro plant with discharge characteristic modelled by Fig 4.3 curve B.

To demonstrate the use of the system load vs duration curve in simplifying step-loading, the system was first scheduled over a period of 24 hours broken into six intervals each of four hours. Three methods of scheduling were compared:

If the only constraints on schedule are those listed above, the system load curve and schedule are illustrated by Fig 4.8.

Schedules for the system with a thermal slack bus plant were the same as that with a hydro slack bus plant. In the hydrothermal example, fuel cost was \$120,827, and in the hydro example, slack bus

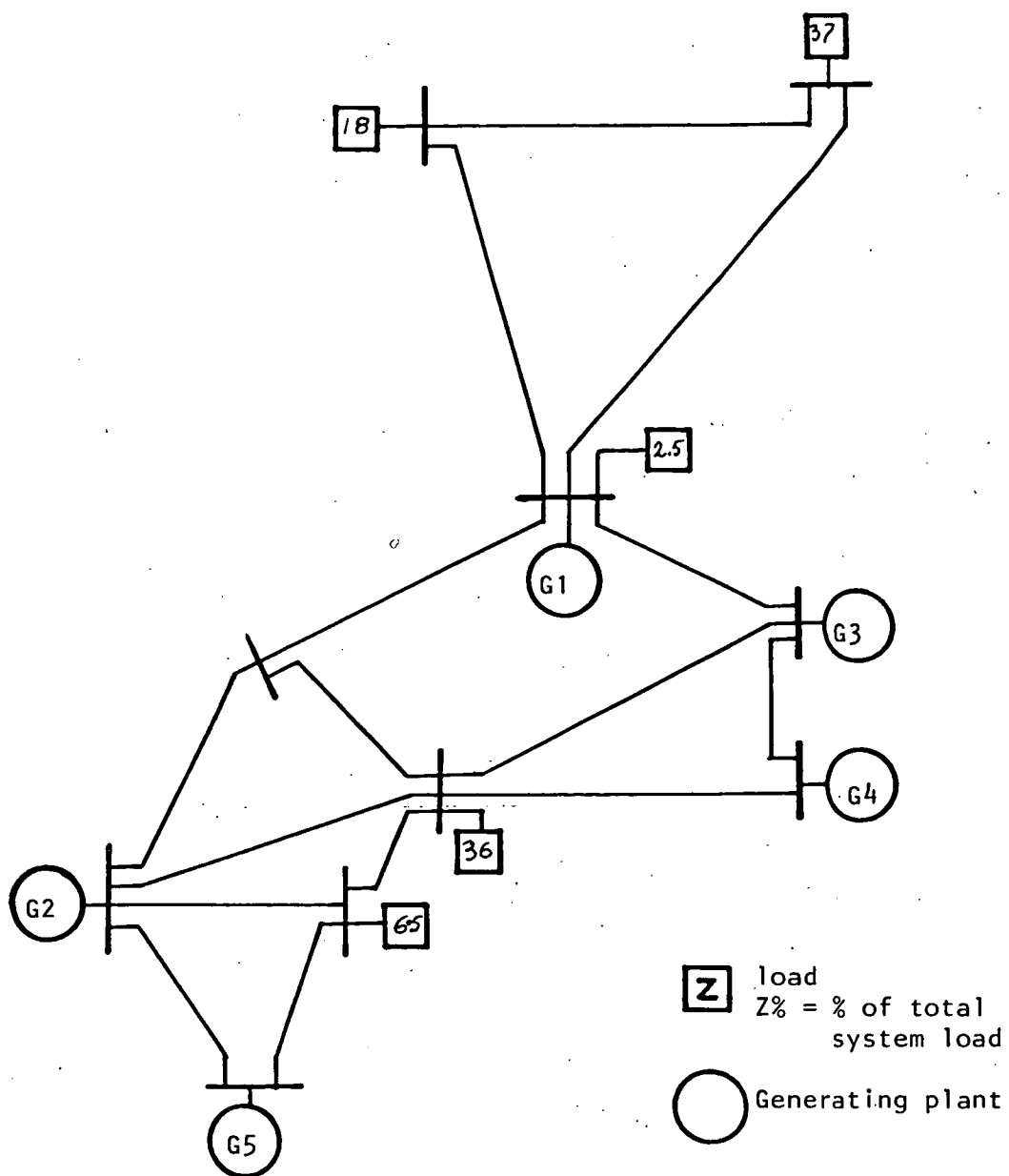


Fig 4.4 5 plant system

Fig 4.5 B-coefficients of the 5-plant system of Fig 4.4
in 10^{-4} MW^{-1} units

	$B_{i, 1}$	$B_{i, 2}$	$B_{i, 3}$	$B_{i, 4}$	$B_{i, 5}$
$B_{1, j}$	1.79008	- .26091	- .44355	.55441	- .85878
$B_{2, j}$		7.04068	.27920	.29749	1.26519
$B_{3, j}$			2.35414	1.03949	.53822
$B_{4, j}$				2.63542	.67813
$B_{5, j}$					6.52986

Fig 4.6 Step-loading modes in the 5-plant system of Fig 4.4

Plant No.	Plant output (MW)		Time in State 1 (hrs)
	State 1	State 2	
1	49.5	76.0	16
2	102.7	128.5	16
3	63.0	82.0	8
4	0.0	21.0	20
5	S l a c k b u s p l a n t		

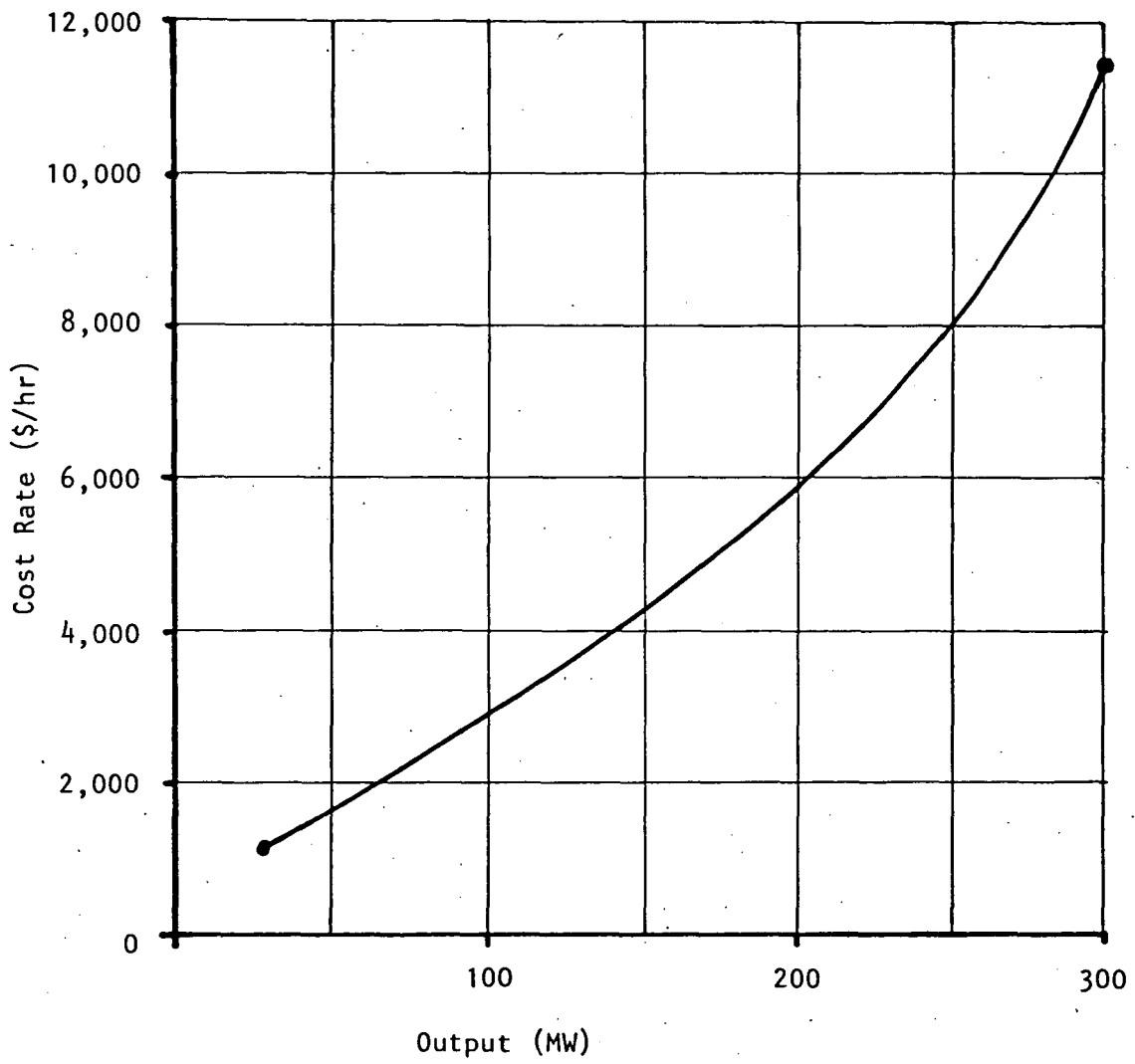


Fig 4.7 Single unit thermal plant cost characteristic

discharge was $22.549_{10}6\text{m}^3$.

When Mantera's [7, 8] constraints of one load transition in each direction for each plant were applied, the best schedules obtainable for both the system with a thermal slack bus plant and that with a hydro slack bus plant were as illustrated in Fig 4.9. In the hydrothermal example, fuel cost was \$122,564, and in the hydro example, slack bus plant discharge was $22.643_{10}6\text{m}^3$.

Applying the principle of one load transition in each direction for each plant to the system load vs duration curve, the system load vs duration curve and best schedules obtainable for both the hydrothermal and hydro systems is shown in Fig 4.10. In the hydrothermal system, fuel cost was \$120,827, and in the hydro example, slack bus plant discharge was $22.549_{10}6\text{m}^3$. As predicted by Section 4.3, if these schedules obtained are then related to the system load curve, the optimum schedule of Fig 4.8 results.

When applied to the system load vs duration curve, the step-loading method of one upward load transition and one downward load transition for each base-load plant yielded a schedule identical to that produced with only the basic constraints of two discharge rates and predetermined total discharge applying. This was done with much less computing, a factor of eight for the five plant system over six scheduling intervals, or if applied over 24 scheduling intervals, a factor of $1.3_{10}16$.

For the system with the single unit thermal slack bus plant, the load vs duration curve method resulted in a thermal fuel cost of \$120,827, while Mantera's method, involving the same amount of computing yielded a cost of \$122,564. The difference, \$1,737 or

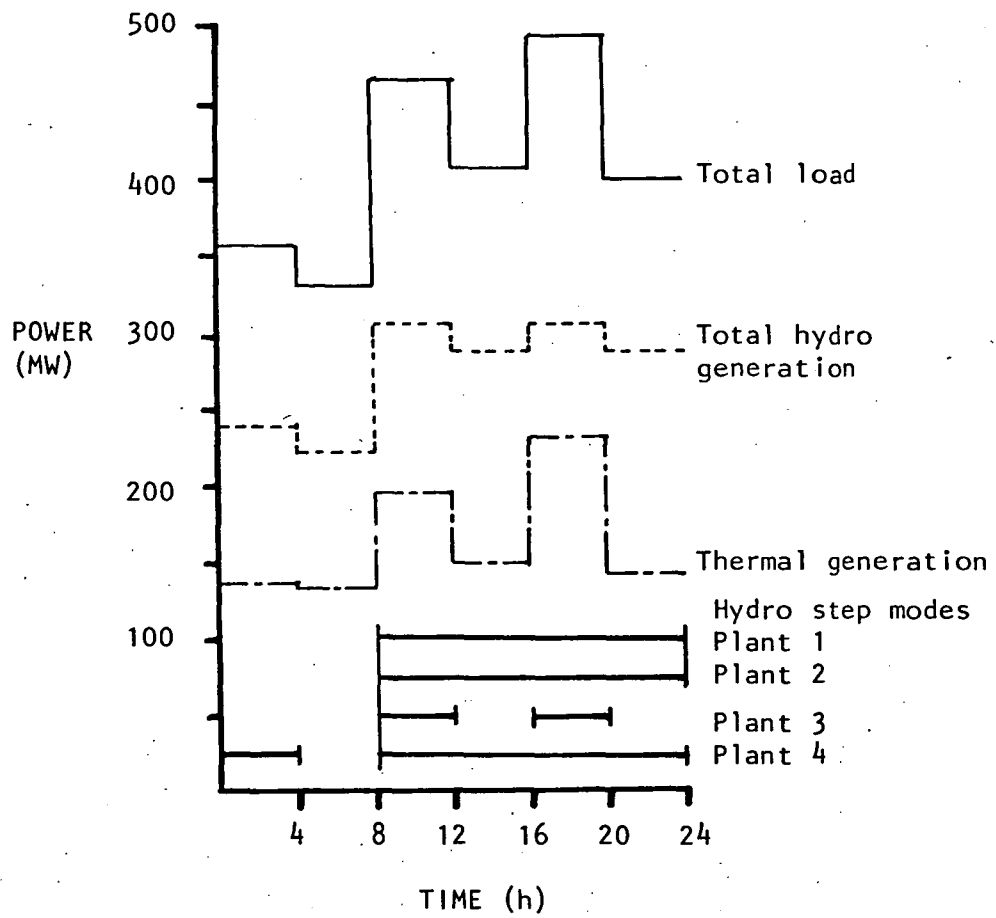


Fig 4.8 Optimum unconstrained step-loading schedule
5 plant system, 6 time intervals

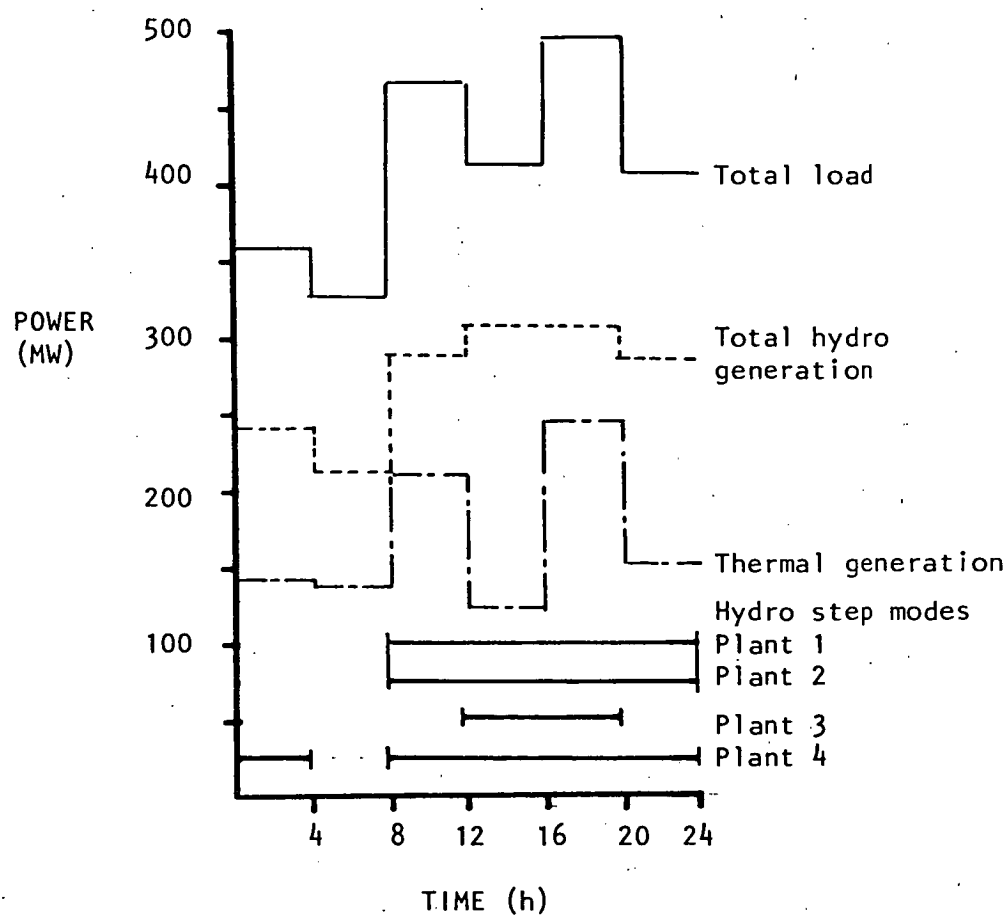


Fig 4.9 Best step-loading schedule obtainable using Mantera's method
5 plant system, 6 time intervals

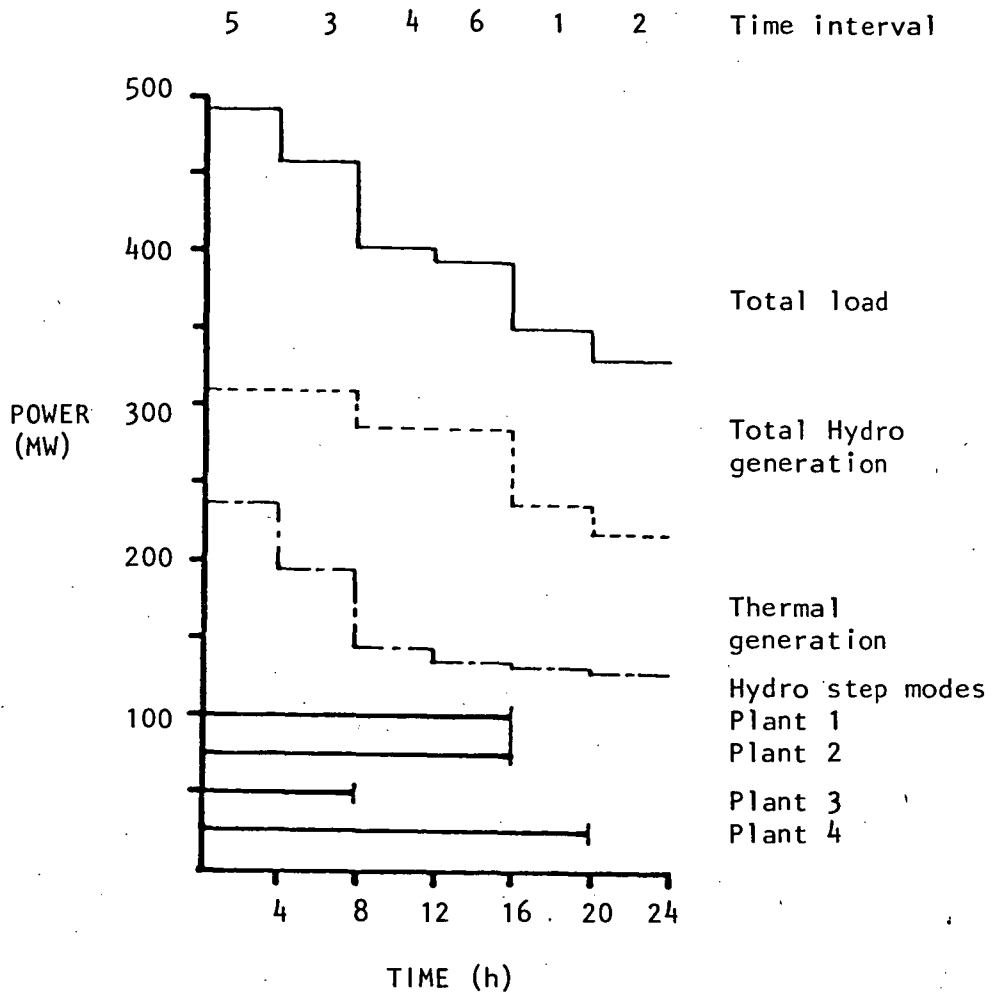


Fig 4.10 Optimum step-loading schedule obtained using a reordered load curve and restricted switching of hydro plants
5 plant system, 6 time intervals

1.4% represents an annual saving of \$634,000. The savings achieved in the example with a five-unit hydro slack bus plant were a rather smaller fraction of total discharge as the plant characteristic was much closer to constant efficiency. The difference between the load vs duration curve method and Mantera's method was a slack bus plant discharge of $94_{10}3\text{m}^3$, or 0.42% of the total slack bus plant discharge.

While the two preceding examples illustrate the use of the system load vs duration curve in simplifying step loading, it is more realistic to split the scheduling period into a larger number of scheduling intervals. Both the hydrothermal and the hydro systems described above have been scheduled on the basis of a one day scheduling period broken into 24 equal intervals each of one hour. The resulting schedules, using both Mantera's method and the load-duration curve method are shown in Figures 4.11 and 4.12. Schedules were identical for the hydro and hydrothermal systems. Thermal plant fuel cost for the load duration curve method was \$122,447 and for Mantera's method \$123,246, a difference of 0.67%.

In the all-hydro system, total discharge from the slack bus plant was $22.546_{10}6\text{m}^3$ for the load duration curve method and $22.589_{10}6\text{m}^3$ for Mantera's method, a difference of 0.19%. The relatively small difference in slack bus plant discharge for the all hydro example reflects the slack bus hydro plant characteristic which is much closer to constant efficiency than the characteristic of a thermal plant.

4.3.2 The Multi-Minima Nature of Step-Loading

Mantera [8] noted that when using a one dimension at a

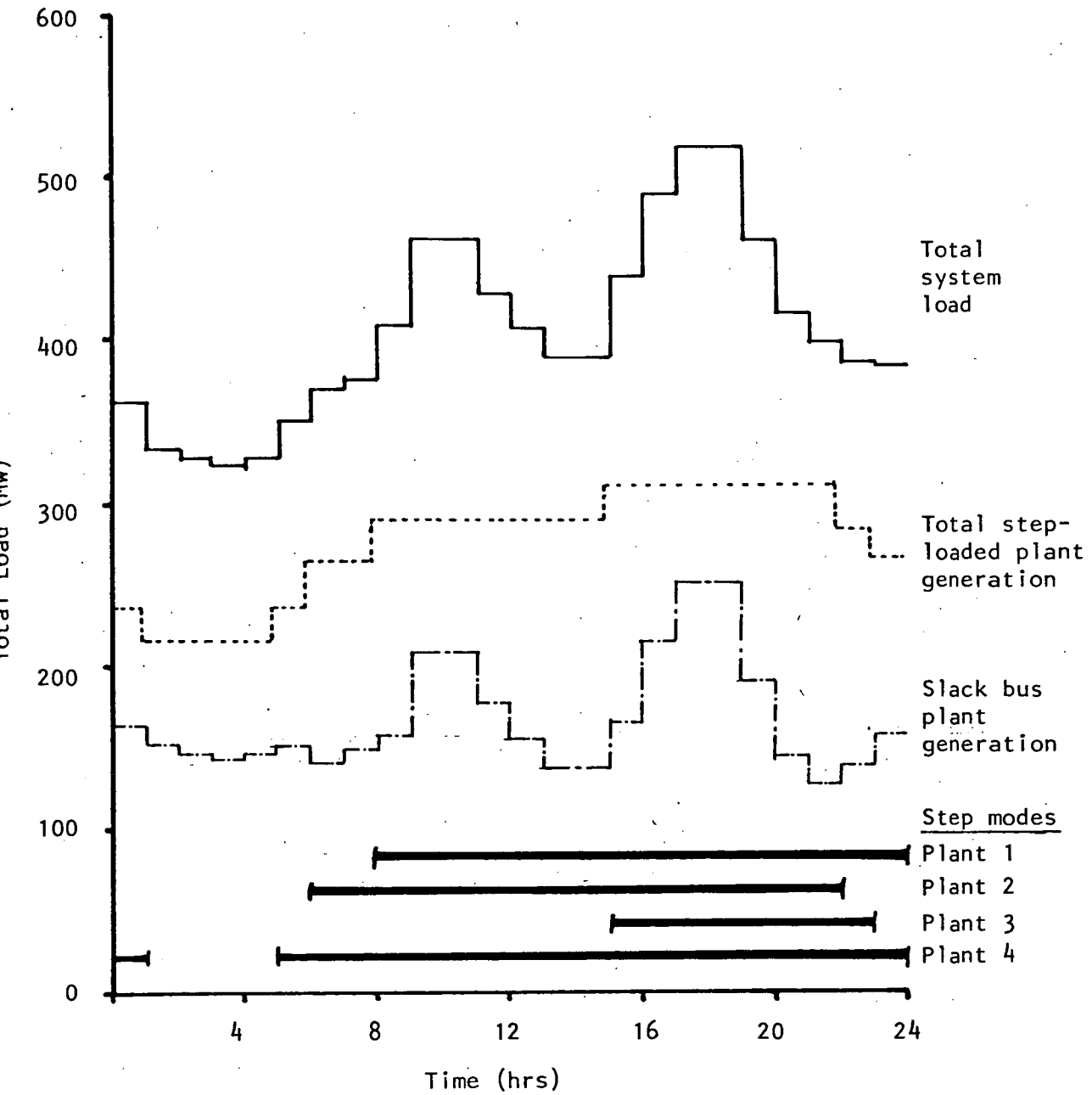


Fig 4.11 Schedule based on Mantera's method for the 5-plant system of Fig 4.4

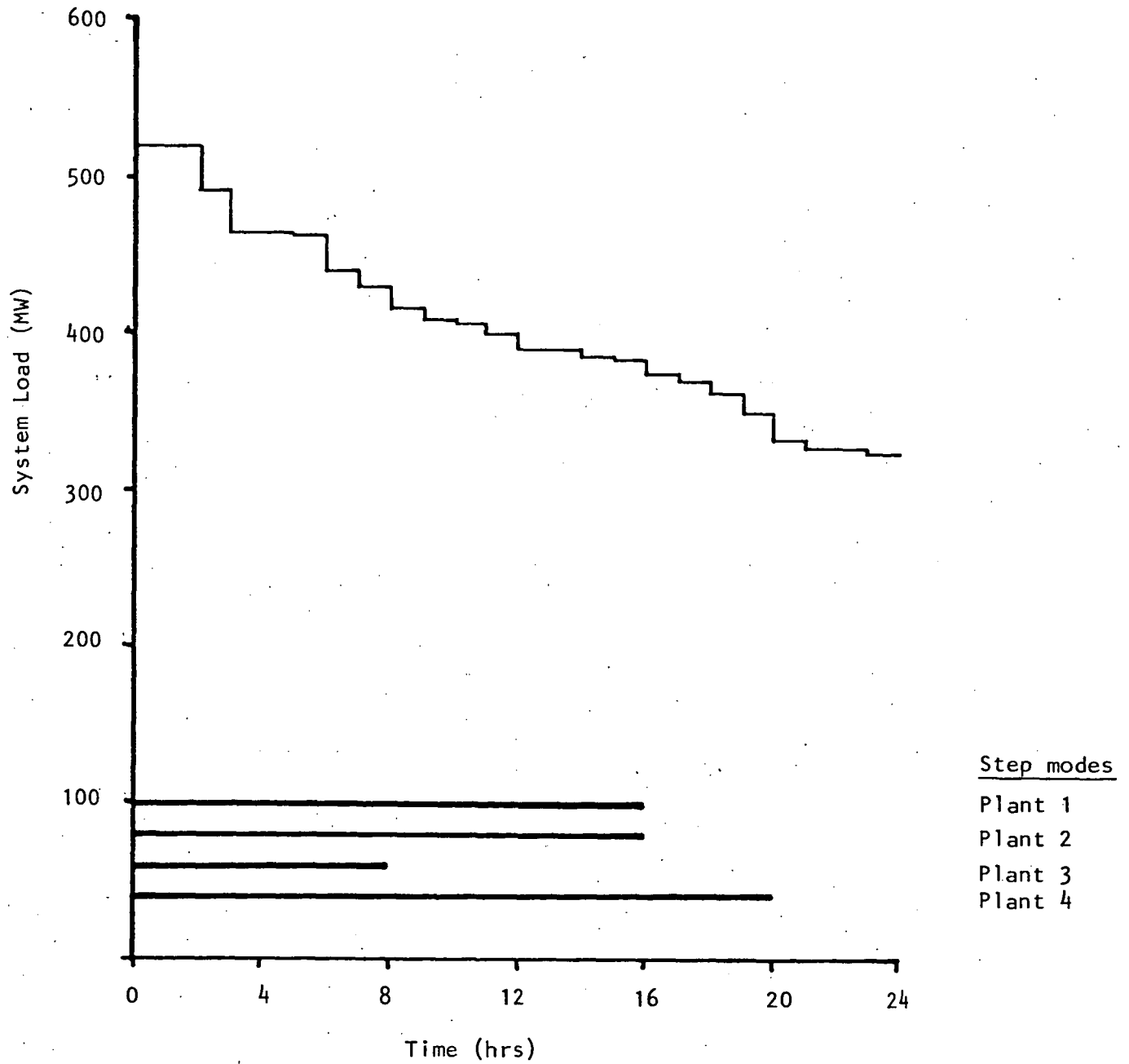


Fig 4.12 (a) Schedule based on the load duration curve for the 5-plant system of Fig 4.4

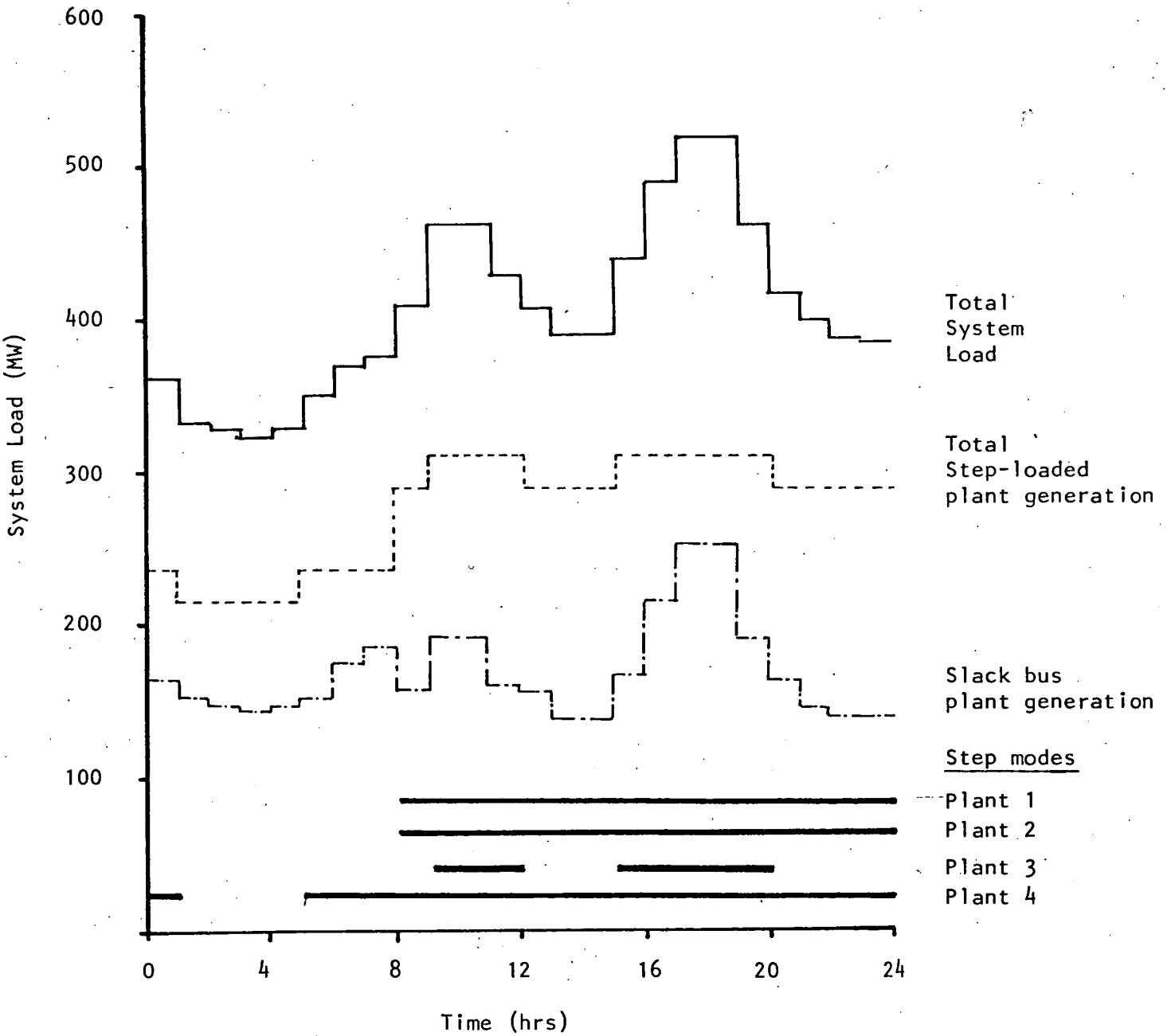


Fig 4.12 (b) Schedule based on the load duration curve for the 5-plant system of Fig 4.4

time search to obtain a step-loading schedule, a number of minima were evident. He did not attempt to locate or eliminate the cause of these spurious minima.

It was decided to exhaustively examine all step loading schedules of the simple 3-plant system of Fig 4.13, allowing one upward transition and one downward transition of load for each of the base-loaded plant. For a one day scheduling period split into 24 intervals of one hour, there are 576 possible schedules. The influence of transmission losses, the shape of the slack bus plant cost characteristic and the shape of the system daily load curve on the number of local cost minima were investigated.

Neither the system transmission losses, which were varied from zero to twice their normal value, nor the shape of the slack bus plant cost characteristic, which was taken first as the single unit thermal plant of Fig 4.7 and then as the five-unit hydro plant of Fig 4.3 curve A, had any significant effect on the number of cost minima.

The shape of the system load curve used in the scheduling process had a substantial influence on the number of cost minima. For the three plant system studied, the normal chronological load curve with two peaks (Fig 4.14(a)) yielded five cost minima, while a single peak load curve (Fig 4.14(b)) gave three minima. There were two minima among schedules based on the system load vs duration curve (Fig 4.14(c)).

A four plant system, which has 13,824 possible step-loaded schedules, was also examined and for the chronological load curve (two peaks), a very large number of load extrema were present. For

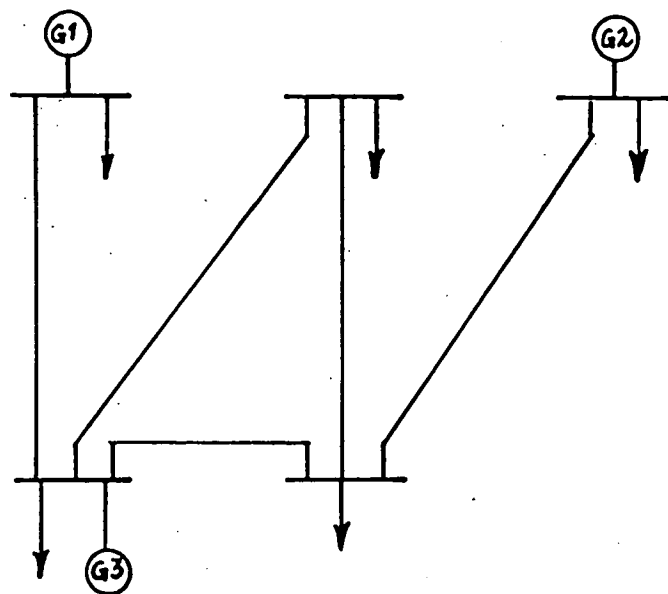


Fig 4.13 3 plant system

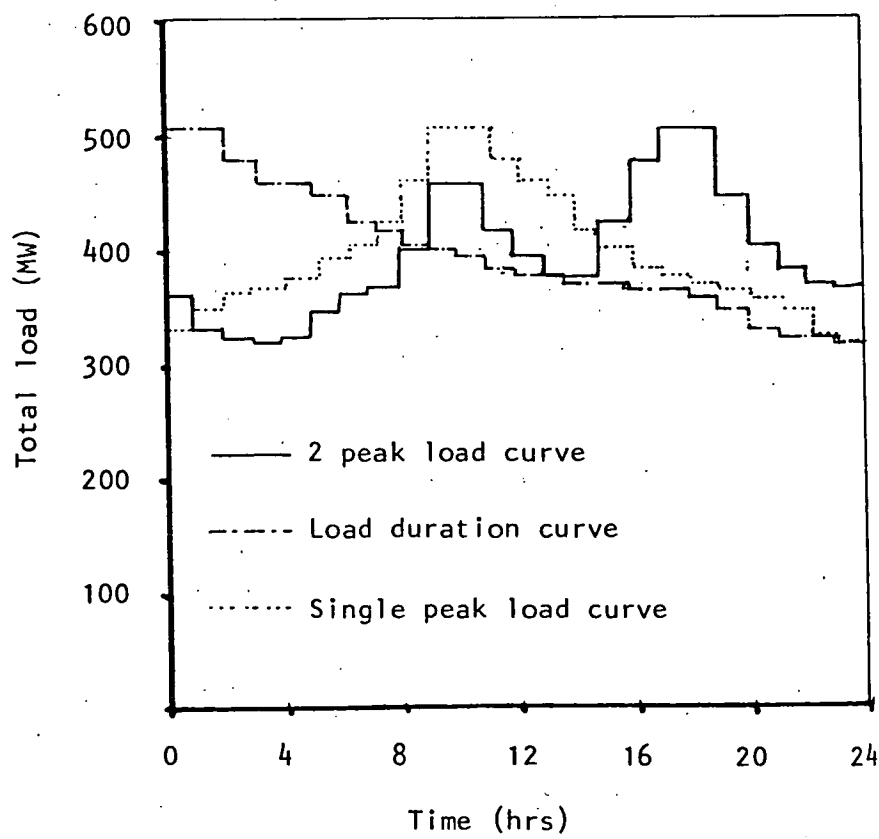


Fig 4.14 Load curves for the 3 plant system of Fig 4.13

schedules based on the system load vs duration curve, a total of eight cost minima were obtained.

Another feature apparent was that each step-loaded plant had at least one load transition time in common with that of one or more other plants. Schedules constrained in this way require significantly less computation. The saving increases as the scheduling period is more finely divided. Load transitions in a particular system may well be constrained further without endangering convergence to the global optimum.

4.4 CASE STUDY

A system comprising 17 hydro plants and one thermal plant was studied. This is fundamentally the same system studied by Mantera [8] and corresponds to a major part of the generation and distribution system operated by the Hydro-electric Commission of Tasmania.

System transmission losses were represented by the B-coefficient loss formula, and to simplify computing, a number of radially connected plants were grouped into one equivalent source, taking radial line losses into account. Figure 4.15 shows the reduced system in which the 18 plants are represented as seven equivalent sources. Transmission loss coefficients, using George's simplified loss formula [5], are tabulated in Fig 4.16.

The 18th plant, the thermal plant, serves as the swing plant. Its cost characteristic is that of Fig 4.7 and upper and lower limits of 270MW and 30MW allow some margin for load forecast errors. To implement this generation constraint, the simple process

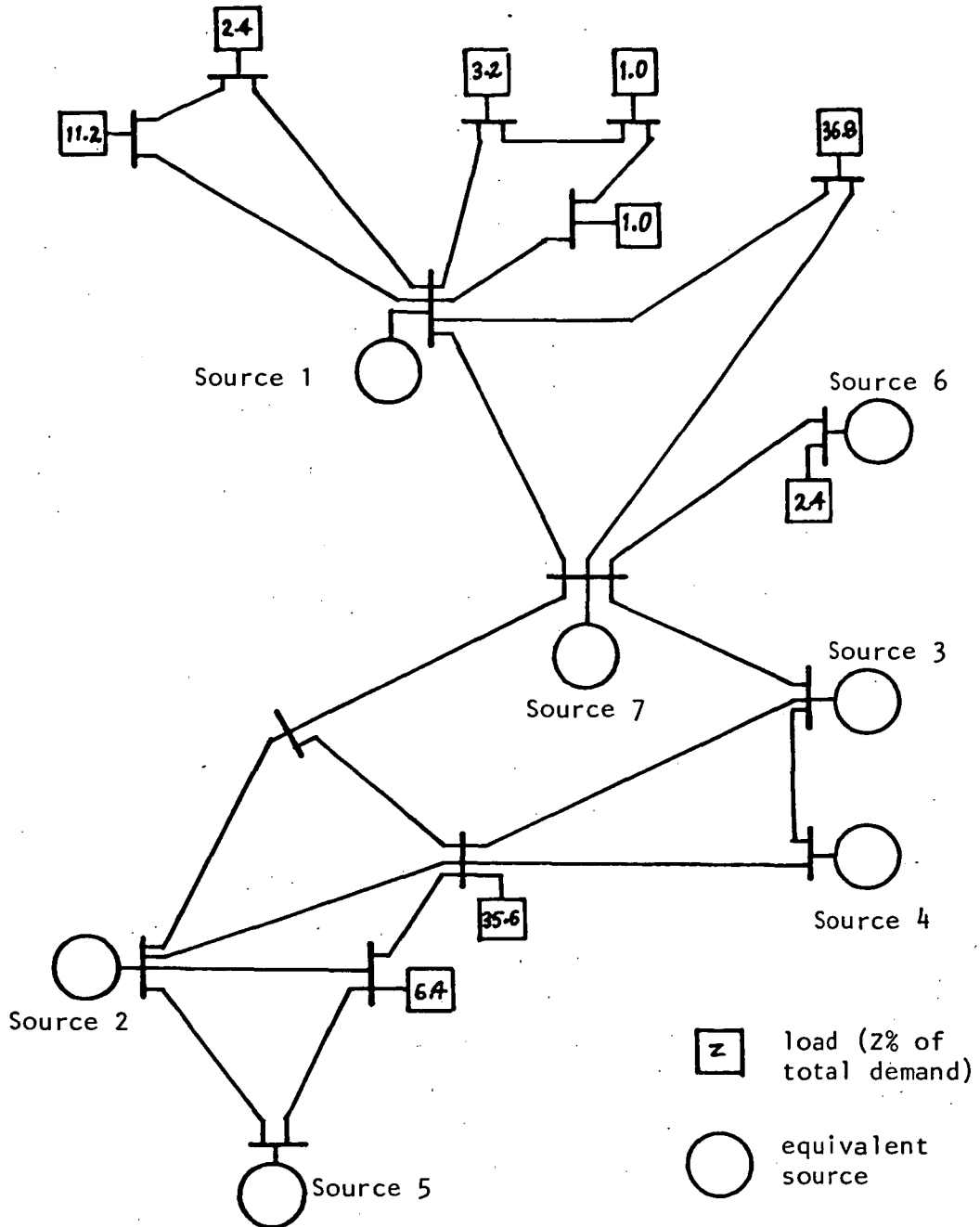


Fig 4.15 18 plant (7 source) system studied [8]

Fig 4.16 B-coefficients of the 18 plant system of Fig 4.15
(in 10^{-4} MW^{-1} units)

	$B_{i, 1}$	$B_{i, 2}$	$B_{i, 3}$	$B_{i, 4}$	$B_{i, 5}$	$B_{i, 6}$	$B_{i, 7}$
$B_{1, j}$	1.79008	-.26091	-.44355	.55411	-.85878	.26533	.35868
$B_{2, j}$		7.04068	.27920	.29749	1.26519	.05199	.14534
$B_{3, j}$			2.35414	1.03949	.053822	-.13064	-.03729
$B_{4, j}$				2.63542	.67813	-.24151	-.14816
$B_{5, j}$					9.82912	-.54588	-.45253
$B_{6, j}$						8.93566	.67158
$B_{7, j}$							1.52986

Fig 4.17 Step loading modes in the 18 plant system of
Fig 4.15

Plant No.	Equiv. Source No.	Plant output (MW)		Time in State 2 (hrs)
		State 1	State 2	
1	1	0.00	9.50	24
2		0.00	40.00	24
3		0.00	40.00	22
4		0.00	29.50	18
5		0.00	82.00	14
6		0.00	56.00	16
7		0.00	18.50	20
8	2	0.00	9.00	24
9		48.40	73.20	10
10		45.30	55.30	15
11	3	27.00	50.00	12
12		14.00	26.00	21
13		0.00	22.00	24
14	4	0.00	21.00	15
15		0.00	9.00	24
16	5	0.00	21.00	20
17	6	36.00	52.50	15
18	7	Slack bus plant		

of adding \$100,000 to the thermal generation cost whenever plant 18 generation was out of range proved adequate. The efficient step-loading modes and hours allotted in each mode for plants 1 - 17 are tabulated in Fig 4.17.

4.4.1 Experimental Results

Since a global search of possible schedules is not feasible because of dimensionality, a gradient method was used, and to eliminate the effect of local minima, the search was started from several initial schedules. Three basic schedules were prepared: a longhand prepared schedule, a schedule based on the chronological load curve and two step-mode changes over the scheduling period, i.e. Mantera's method, and a schedule based on the load vs duration curve for the scheduling period. The two computerised schedules were prepared on a desk-top computer programmed in basic.

The results of the three scheduling methods applied to the 18 plant system are summarised as follows:

Longhand prepared schedule

Operating cost of plant 18 for the 24 hour scheduling period was \$87,795. In preparing this schedule, no transmission losses were calculated, and the aim was to arrange the generation of the 17 step-loaded plants such that plant 18 operated at near-constant load, so achieving most efficient operation if transmission losses are not a function of generation pattern. The schedule, along with the system load curve, is shown in Fig 4.18. Preparation time was in excess of two manhours. Transmission losses were 650.61 MWhr.

Schedule based on the chronological load curve

Operating cost of plant 18 for the 24 hours scheduling period was \$87,697. The schedule, which was calculated in about ten minutes on a desk top computer, is shown in Fig 4.18. The operating cost of plant 18, and therefore of the system was \$98 less than the longhand schedule, and all of this saving was due to a 1.2% decrease in transmission losses to 642.88 MWhr. The efficiency of operation of plant 18 actually fell by 0.14%.

Schedule based on the load vs duration curve

Operating cost of plant 18 for the 24 hours scheduling period was \$87,560. This schedule, having the same preparation time as that based on the chronological load curve is shown, in relation to the system load vs duration curve, in Fig 4.18 and in relation to the system load curve in Fig 4.19.

The operating cost of plant 18, and therefore of the system, was \$137/day less than that of the schedule based on the chronological load curve, with the majority of this improvement due to a 0.066% rise in efficiency of plant 18. There was also a 0.23% decrease in transmission losses to 641.40 MWhr.

Overall, the operating cost was \$235/day less than the longhand schedule, with a 1.42% or 9.21 MWhr decrease in transmission losses and a .072% decrease in plant 18 efficiency.

If plant 18 had been a multi-unit hydro plant with an output characteristic similar to Fig 4.3, i.e. almost constant efficiency in mid-range, rather than the nonlinear characteristic, Fig 4.7, of a single unit thermal plant, the full 9.21 MWhr saving in transmission losses would have passed directly into a saving of water.

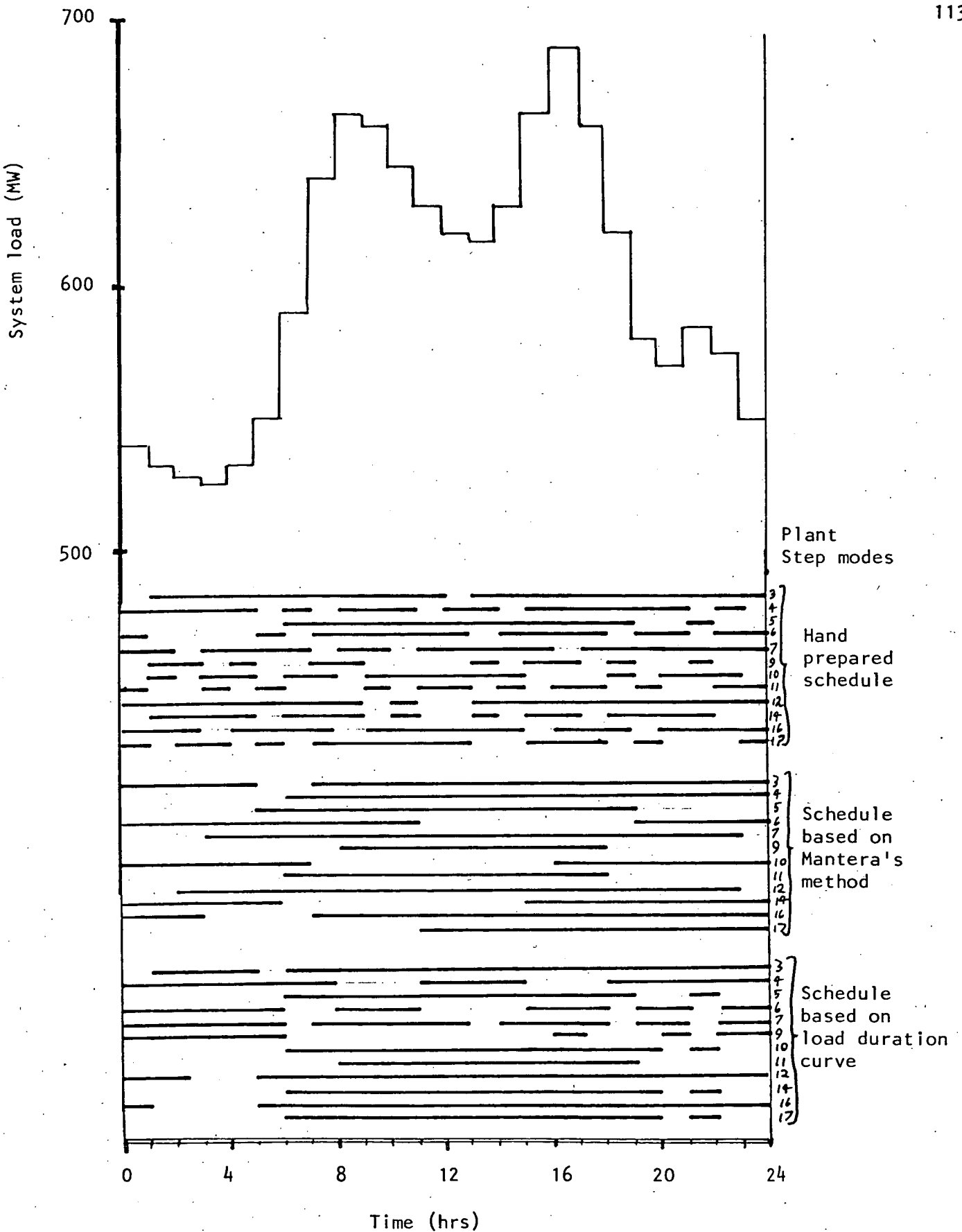


Fig 4.18 Step-loading schedules for the 18-plant system of Fig 4.15

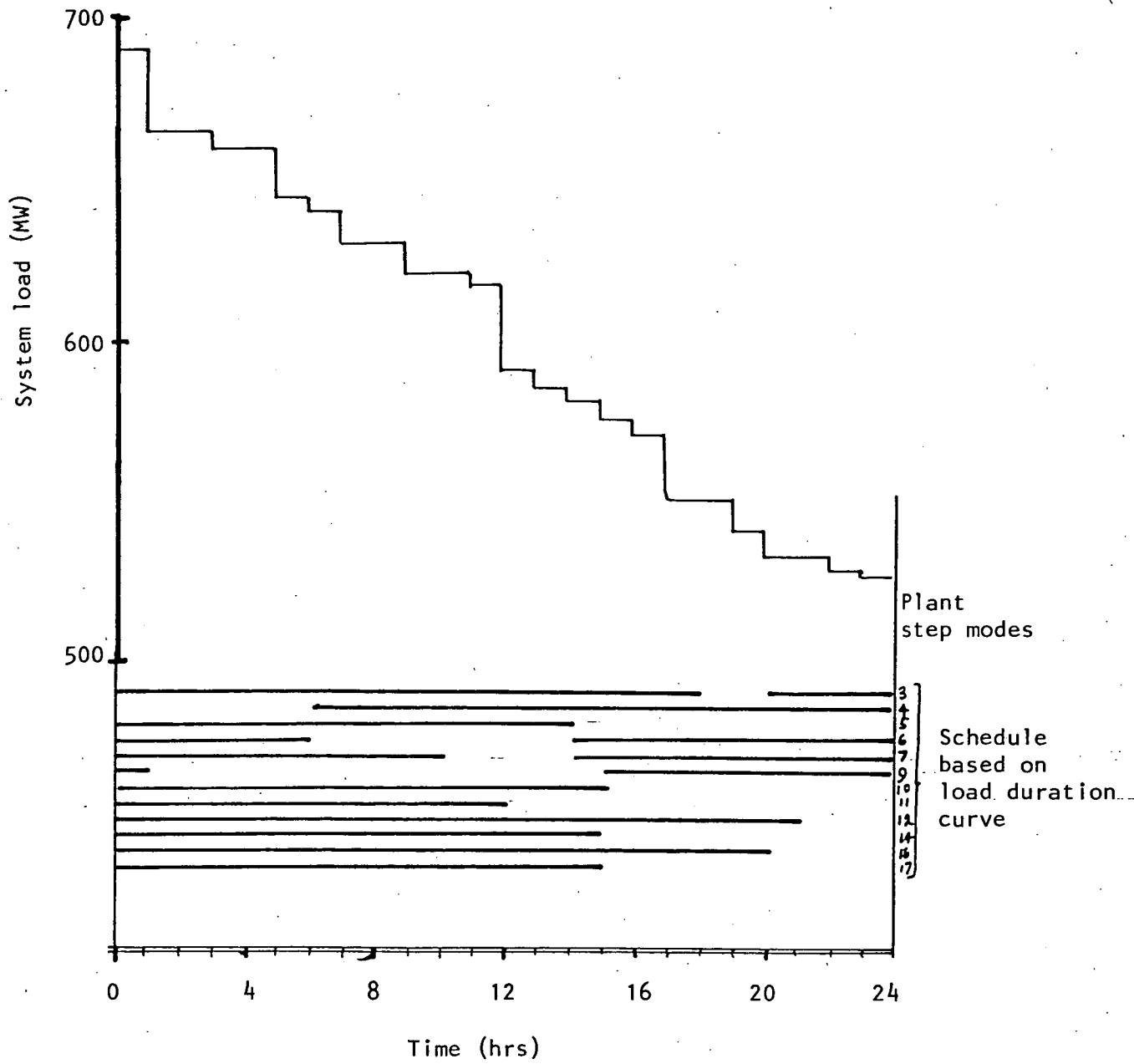


Fig 4.19 Step loading schedules for the 18-plant system of Fig 4.15

Both in schedules based on the chronological load curve and those based on the load vs duration curve, the multi-minimal nature of the scheduling problem was evident, and it was necessary to re-run the search procedure from several starting points to obtain the true optimum.

4.5 EFFECTS OF HEAD VARIATIONS

The effects of plant head variations over the scheduling period have so far been ignored. This is justified where plant head is high or the storage is large.

Low head plants, fitted in most cases with Kaplan turbines which have almost constant efficiency over their operating range are well suited to run-of river operation, with the plant discharge equal at all times to the reservoir inflow from both upstream plants and external sources. They can, therefore, be combined with their upstream neighbours such that a number of cascaded low-head plants with one upstream high or medium head neighbour can be replaced for analysis by a single equivalent plant which is step-loaded in accordance with a composite characteristic curve.

Treating low head plants in this way, the effects of head variations need be considered only for medium head plants having storages of only a few days capacity.

While the output per unit flow (specific output) of a hydro plant changes in almost direct proportion to head, the discharge rates corresponding to most efficient loads are almost unchanged. Head variations of those plants in which the effect is significant can therefore be accounted for as part of the step-

loading algorithm with a minimum of added computing simply by stepping between most efficient discharge rates rather than loads, and computing loads from these discharge rates in each time interval.

This is not expected to alter the optimum step-loaded schedule, and when applied to the 18 plant system studied in 4.4, the only effect for 2% head variations in a number of plants was to alter the generation and therefore operating cost of the thermal plant, leaving the discharge schedules of plants 1 - 17 unchanged.

Minimum head constraints can be handled using a penalty factor approach or simple schedule rejection, while maximum head constraints are modelled by an increase in flow without a corresponding increase in generation, i.e. spilling into the downstream storage if one exists.

In view of the result for the 18 plant system, a step loading schedule not considering head variations followed by a simple check of storage levels in each time interval is considered to be adequate.

4.6 DISCUSSION

A step-loading technique developed in this chapter requires all but one plant of an all-hydro system, or all hydro plants in a hydro-thermal system, to be step-loaded between two efficient discharge rates or to be operated as run-of-river. Step-loading parameters for each plant are determined from the total discharge allotted by the longer-term schedule and the characteristic curve of the plant, or, for a cascaded group of plants, by the characteristic curve of an equivalent plant.

To reduce computing, it was proposed that the system load vs duration curve rather than the chronological load curve be used, and, based on this curve, that each plant be restricted to two changes in its discharge rate. This method was tested on both five-plant and 18-plant systems.

A useful spin-off from using the load vs duration curve as the schedule basis is that a set of guidelines for synchronising and shutting down units is produced. Referring to Fig 4.10 we obtain, for the five-plant system the following:

Plant	No. of units on line			
	1	2	3	4
Load < 350	1	3	2	0
$350 \leq \text{Load} < 384$	1	3	2	1
$384 \leq \text{Load} < 430$	2	4	2	1
$430 < \text{Load}$	2	4	3	1

Optimal step-loading schedules based on the load vs duration curves are characterised by each plant having at least one, and more usually both step-points in common with that of another plant. An explanation of this is that the optimum schedule is one of near-constant load operation of the slack plant. When a large unit is shut, a smaller unit or units must be synchronised to minimise the step change in total step-loaded generation and therefore in slack plant generation since the load vs duration curve is essentially smooth. This may lead to the concept of a maximum allowable step size in total step-loaded plant generation for a particular system, allowing unsuitable schedules to be swiftly eliminated without recourse to

loss calculation, but this has not been investigated.

While examples in this chapter utilised the simplified loss formula to represent transmission losses, an actual application would, as a minimum, use the generalised loss formula [3, 4]. These loss coefficients may be swiftly computed by optimally ordered triangular factorisation and Newton's method [9], incorporating the effects of any constant impedance loads, tap-changers and phase shifters. For greater accuracy, a similar set of B-coefficients could be computed [9] relating transmission losses to net power inflow at each system busbar rather than generation alone. A further alternative may be to abandon the B-coefficient loss formula in favour of fixed Jacobian load flows [10] or similar.

The step-loading technique of this chapter may be used as a stand-alone scheduling method or used as a base for further refinement. Later chapters utilise step loading to provide hydro unit commitment.

4.7 REFERENCES

- [1] Adams, R.L. "Economic load flow control" 1975 Residential School in Power System Electrical Engineering, University of Tasmania, Hobart (1975).

- [2] Dillon, T.S. "Problems of optimal economic operation and control of integrated (hydrothermal) and thermal power systems". Ph. D. thesis. Department of Electrical Engineering, Monash University (1974).

- [3] Early, E.D., Watson, R.E. "A new method of determining constants for the general transmission loss equation". Trans. AIEE Pwr. Appar. Sys. v. 74 (1956) pp 1417-1423.

- [4] Early, E.D., Watson, R.E., Smith, G.L. "A general transmission loss equation". Trans. AIEE Pwr. Appar. Sys. v. 74 (1955) pp 510-520.

- [5] George, E.E. "Intrasystem transmission losses". Trans. AIEE v. 62 (1944) pp 153-158.

- [6] Jones, G.V., Norton, J.P. "Hydrothermal optimisation by refinement of step-loading schedules". IFAC Symposium, Melbourne 1977.

- [7] Mantera, I.G.M. "Optimal daily load scheduling of hydro-electric systems by means of step-loading technique" Unpublished.
- [8] Mantera, I.G.M. "Optimal load scheduling of hydro-electric power stations" Ph. D. Thesis, Electrical Engineering Dept., University of Tasmania (1972).
- [9] Meyer, W.S. "New computational techniques for the power flow and economic dispatch problems of electric utility engineering". Ph. D. Thesis, University of Minnesota (1969).
- [10] Nabona, N., Freris, L.L. "Optimisation of economic dispatch through quadratic and linear programming". Proc. IEE v. 120 No. 5 (1973).

5. REFINEMENT OF THE STEP-LOADING SCHEDULE BY RELAXATION USING A DECOMPOSITION TECHNIQUE

The system model, summarised by Fig. 5.1, includes

- (a) Thermal plants represented by their cost characteristics,
- (b) An electrical network represented by load-flow equations,
- (c) Hydro plants represented by their head- and discharge-rate-dependent characteristics,
- (d) Hydro networks represented by sets of flow difference equations and,
- (e) Bounds on equipment ratings and operating conditions of both electrical and hydro networks.

The "optimal" schedule of a hydro-thermal power system may be computed either by considering separated hydro and electrical/thermal subproblems with independent goals, or by considering the integrated problem.

Using separated subproblems, [9, 10] the hydro network operation is optimised relative to an appropriate performance index for generated hydro power, such as maximum energy output with fixed total water use by each plant, then the corresponding best operation of the thermal plants and electrical network is determined. Some success may be obtained with this decoupled method in systems having a low proportion of hydro-generation, widely separated from the thermal generation/load regions.

The integrated approach computes the optimal solution of the entire system. Examples of this are the methods of Kirchmayer,^[9]

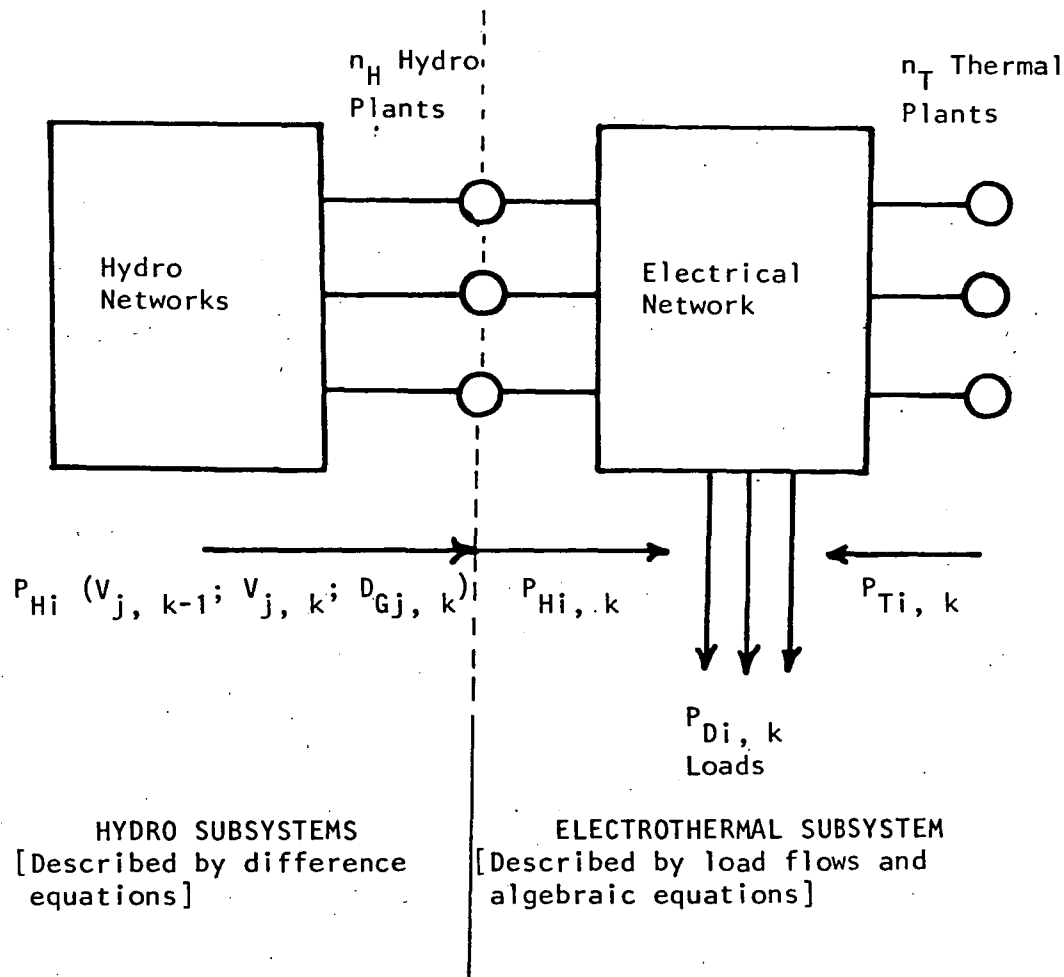


Fig 5.1 The hydrothermal power system

Bernholtz and Graham, [2] Fukao, Yamazaki and Kimura, [8] Bonaert, El-Abiad and Koivo [3] and Dillon [5, 6, 7]. References [2], [8] and [9] use a quadratic loss formula in place of a complete electrical system model and so do not account for reactive VA, equipment ratings or stability and security limitations. Reference [3] uses a load flow model of transmission loss, but permits only discrete increments in hydro plant generation, as it is an extension of [2].

Because the hydrothermal scheduling problem is variational, an integrated solution involves treating a problem which is infinite-dimensional in the time-domain as finite-dimensional. The result of this is that for a system of realistic size, there are a very large number of state variables [5] and inequality constraints, giving rise to dimensionality difficulties. Alternatively, the integrated problem may be solved via a sequence of lower-dimensional problems, alternately rescheduling only hydro and only thermal plants, both with reference to the same goal. This is the technique of decomposition used in this thesis.

5.1 DECOMPOSITION OF THE HYDROTHERMAL POWER SYSTEM INTO SUB-SYSTEMS

To apply a decomposition technique, the system must contain groups of variables which, although explicitly related to one another within a group, have no explicit relations between variables of different groups. The sets of load-flow variables $\{P_k, Q_k, E_k, \phi_k, P_{kj}, \tau_{kj}, T_{kj}\}$ do not have cross-product terms in different time intervals k , either in the objective function or in

the constraints, and no explicit constraints relate these sets at different values of k .

The hydro plant characteristics relate the above sets to the hydro variables $\{V_k, D_{Gk}, D_{Bk}, D_{Sk}\}$ but, if an improved solution only is required rather than obtaining the optimum in one step, this link can be ignored and the load flow variables may be treated as separable from the hydro variables within each step of the optimisation. The sets of hydro variables for two hydraulically separate river systems have no cross-product terms, and are not related by any explicit constraints.

The system may thus be decomposed into an electrothermal subsystem, described by equations (3.1) to (3.13) and solved separately in each time interval k , and a group of hydro subsystems, one for each river system, described by equations (3.14) to (3.22). That is, m static subsystems, one for each time interval, and l dynamic subsystems, one for each river system. This is illustrated by Fig. 5.2.

5.2 THE PROPOSED METHOD OF SCHEDULE REFINEMENT

The procedure refines an initial step loading hydro schedule, employing optimal load flow computations to schedule thermal generation, VAR allocation and regulator settings in compliance with constraints on the electrothermal subsystems, alternating with a suitable hill-climber using the OLF results to improve the hydro schedule.

The family of techniques known as optimal load flows, originating about eighteen years ago, [4] are now reliable and

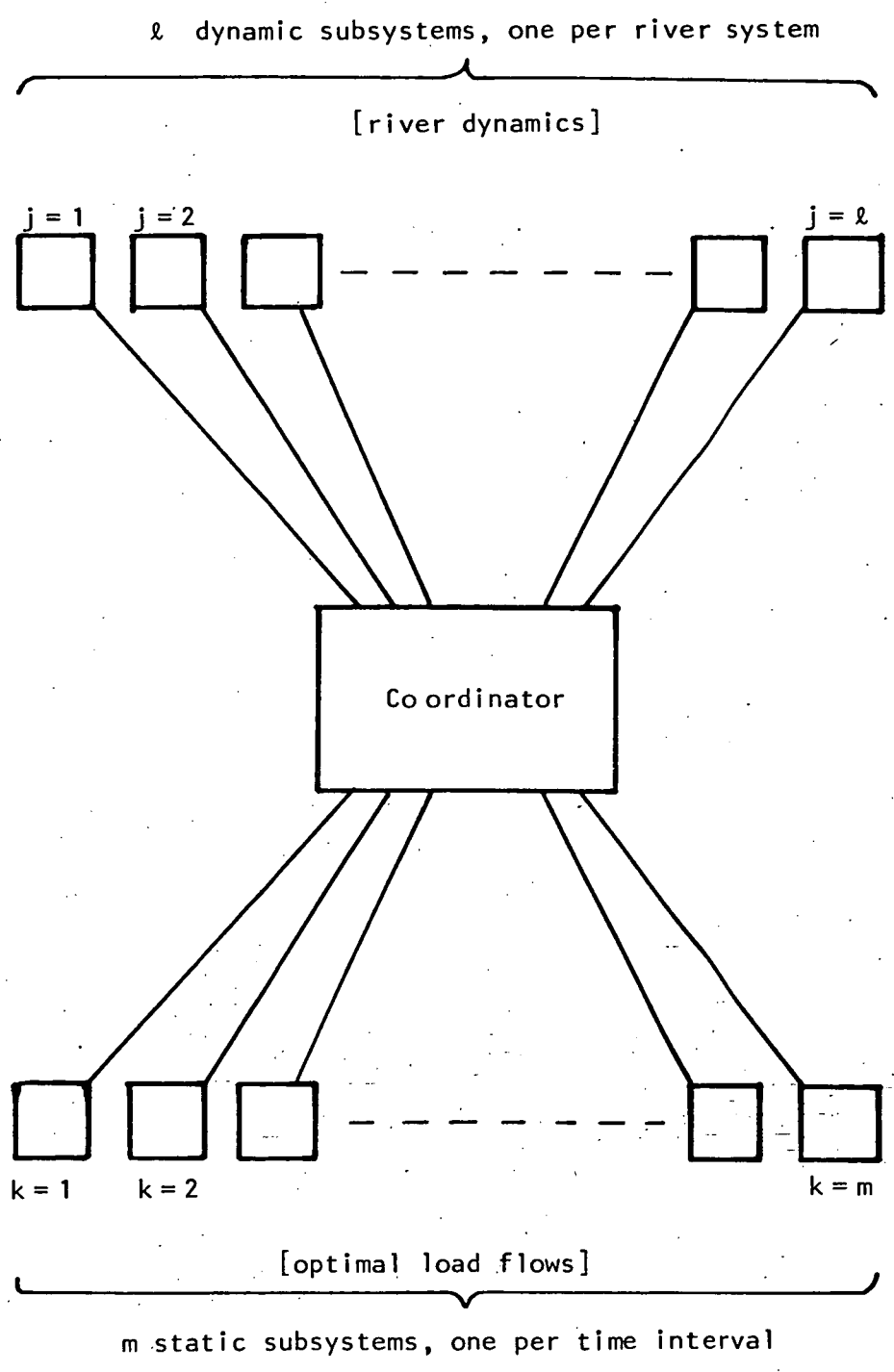


Fig 5.2 Sectioning of the integrated problem

efficient, providing gradient information suited to use by the hill-climber used to improve the hydro schedule. These optimal load flow techniques are generally based on the Newton load flow algorithm [11, 12] which is familiar to system controllers, thus facilitating both implementation and initial acceptance of schemes such as the one proposed. Several alternative OLF methods are discussed in Section 5.3 and the method chosen for tests is described in detail by Section 5.4.

The fundamental need in hydro-thermal system scheduling is for a method which will run in an acceptable computing time on a small to medium sized computing system, and preferably able to be run on a minicomputer based system. This imposes the restriction that the hill-climber chosen for hydro-schedule refinement must converge swiftly in the first few steps, and also highlights a major advantage of the proposed scheme: if it is necessary to halt the iteration process at any stage, then a feasible and improved schedule will always result.

5.3 THE CHOICE OF A HILL-CLIMBER FOR CO ORDINATION OF THE ELECTROTHERMAL AND HYDRO SUBSYSTEMS

The choice of co ordination algorithm is dictated by a need to keep the number of OLF computations small. In particular the hill-climber must be judged on its performance over a small number of steps. The shape of the cost vs discharge-rates surface must be exploited and if second derivatives are to be used, the extra OLF computations needed must be justified by the increase in hill-climbing speed. This requirement of very few steps rules out

such methods as PARTAN and the various simplex techniques.

The hourly discharge rates \underline{u}_k ; $k = 1, 2, \dots, K$ determining the power flows at the co-ordination busbars must give the specified water use

$$\sum_{k=1}^K \underline{u}_k = \underline{q} \quad \dots (5.1)$$

The minimum total cost, f , may be found by adjoining the total discharge constraints with Lagrange multipliers $\underline{\lambda}$, giving

$$L = f + \underline{\lambda}^T \cdot \left(\sum_{k=1}^K \underline{u}_k - \underline{q} \right) \quad \dots (5.2)$$

then setting

$$\nabla_{\underline{u}_k} L = \underline{g}_k + \underline{\lambda} = \underline{0}, \quad k = 1, 2, \dots, K \quad (5.3)$$

Where \underline{g}_k is the gradient of f with respect to \underline{u}_k . At the optimum, the cost vs discharge rate gradients are the same for each time interval.

In principle, the optimum can be found by iterating $\underline{\lambda}$, solving (5.3) for \underline{u}_k at each time and adjusting $\underline{\lambda}$ until (5.1) is satisfied. In practice, efficient adjustment of the discharge rates requires second-derivative information obtained by further OLF solutions for small changes in co-ordination powers. Even then, to avoid iterative adjustment of \underline{u}_k , inversion of the Hessian matrix for each k is required at every trial $\underline{\lambda}$ unless some simplifying assumption can be made. A simplified second-order method which calculates $\underline{\lambda}$ in one step without matrix inversion is described later.

Alternatively, λ need not be found explicitly, total cost being minimised by bringing the "constrained gradients"

$$\underline{g}_{kj} = \underline{g}_k - \underline{g}_j; k = 1, 2, \dots, K \quad \dots (5.4)$$

with j any value from 1 to K , to zero by adjusting all hourly discharges except \underline{u}_j freely, the change in \underline{u}_j being determined by the constraints (5.1). The choice of technique is narrowed by the need to limit the number of OLF computations and to exploit known characteristics of $f(\underline{u})$. In particular, f is the sum of independent costs $f_k(u_k)$ near quadratic and with strongly diagonal-dominant Hessians. Simple gradient and second-derivative hill-climbers taking advantage of these features can be devised.

5.3.1 Gradient Hill Climbers

In the gradient methods, the step-length is calculated from second-derivative information gained from an exploratory gradient step. Collecting constrained gradients \underline{g}_{kj} over the whole scheduling period into a vector \underline{g}_c and expanding $f(\underline{u} + \delta \underline{u}_c)$ up to quadratic terms in its Taylor series

$$f^1 = f(\underline{u}^0 + \delta \underline{u}_c) \approx f^0 + \underline{g}_c^0 \cdot \delta \underline{u}_c + \frac{1}{2} \delta \underline{u}_c^T \cdot H_c^0 \cdot \delta \underline{u}_c \dots (5.5)$$

where H_c^0 is the Hessian corresponding to \underline{g}_c at the current discharge schedule \underline{u}^0 . At the end of the gradient step

$$\underline{g}_c^1 = \underline{g}_c^0 + H_c^0 \cdot \delta \underline{u}_c = (I - a H_c^0) \cdot \underline{g}_c^0 \quad \dots (5.6)$$

If the step length, a , is chosen to minimise f^1 , the new gradient is orthogonal to the search direction, so the optimum step size, a^* , is given by

$$a^* = \underline{g}_c^{0T} \cdot \underline{g}_c^0 / \underline{g}_c^{0T} \cdot H_c^0 \cdot \underline{g}_c^0 \quad \dots (5.7)$$

The denominator can be written in terms of either the gradient $\hat{\underline{g}}_c^0$

or the cost \hat{f}^0 at the end of an exploratory gradient step of size \hat{a} .

Using the gradient, (5.6) gives $H_c^0 \underline{g}_c^0$ and

$$a^* \approx \hat{a} \underline{g}_c^{0T} \cdot \underline{g}_c^0 / \underline{g}_c^{0T} \cdot (\underline{g}_c^0 - \hat{\underline{g}}_c^0) \quad \dots (5.8)$$

Alternatively [1] from the Taylor series

$$\underline{g}_c^{0T} \cdot H_c^0 \cdot \underline{g}_c^0 = 2 (\hat{f}^0 - f^0 + \hat{a} \cdot \underline{g}_c^{0T} \cdot \underline{g}_c^0) / \hat{a}^2 \quad \dots (5.9)$$

so that

$$a^* \approx \hat{a}^2 \underline{g}_c^{0T} \cdot \underline{g}_c^0 / 2 (\hat{f}^0 - f^0 + \hat{a} \cdot \underline{g}_c^{0T} \cdot \underline{g}_c^0) \quad \dots (5.10)$$

The choice between these two expressions for a^* depends on their relative susceptibility to quadratic approximation errors and the time consumed in computing gradients as well as the time taken in executing the exploratory OLF. Rather than minimising f directly, gradient searches can minimise $\underline{g}_c^T \cdot \underline{g}_c$ equally conveniently. The optimum must then be zero so providing a measure of progress, although not in terms of the cost function. Since H_c^0 is symmetric, from (5.6).

$$\frac{\partial}{\partial a} (\underline{g}_c^{1T} \cdot \underline{g}_c^1) \approx 2a \underline{g}_c^{0T} \cdot H_c^{0T} \cdot H_c^0 \cdot \underline{g}_c^0 - 2 \underline{g}_c^{0T} \cdot H_c^0 \cdot \underline{g}_c^0 \quad \dots (5.11)$$

so that the step size minimising $\underline{g}_c^{1T} \cdot \underline{g}_c^1$ is

$$a^* \approx \underline{g}_c^{0T} \cdot H_c^0 \cdot \underline{g}_c^0 / \underline{g}_c^{0T} \cdot H_c^0 \cdot \underline{g}_c^0 \quad \dots (5.12)$$

which like (5.7) can be rewritten

$$a^* \approx \hat{a} \underline{g}_c^{0T} \cdot (\underline{g}_c^0 - \hat{\underline{g}}_c^0) / (\underline{g}_c^0 - \hat{\underline{g}}_c^0)^T \cdot (\underline{g}_c^0 - \hat{\underline{g}}_c^0) \quad \dots (5.13)$$

or

$$a^* = \frac{2 (\hat{f}^0 - f^0 + \hat{a} \cdot \underline{g}_c^0 \cdot \underline{g}_c^0)}{(\underline{g}_c^0 - \hat{\underline{g}}_c^0)^T \cdot (\underline{g}_c^0 - \hat{\underline{g}}_c^0)} \quad \dots (5.14)$$

Tests of the four gradient hill-climbers described by equations (5.8), (5.10), (5.13) and (5.14) are presented in section 5.4.

A more symmetrical function of gradients, whose minimisation satisfies (5.3) while treating all time intervals identically is

$$S = \sum_{k=1}^K \sum_{j=k+1}^K \{(\underline{g}_k - \underline{g}_j)^T \cdot (\underline{g}_k - \underline{g}_j)\} \quad \dots (5.15)$$

Minimising S by gradient steps

$$\underline{g}_r = \underline{g}_r^0 - a_r H_r^0 \cdot \underline{g}_r \quad r = 1, 2, \dots, K \quad \dots (5.16)$$

in the unconstrained gradients, the optimal step lengths are given by

$$\begin{aligned} \frac{\partial S}{\partial a_r} &= 2 \sum_{k \neq r}^K \{ -\underline{g}_r^0 \cdot (H_r^0 \cdot \underline{g}_r^0 - \underline{g}_k^0 + a_k \cdot H_k^0 \cdot \underline{g}_k^0) \\ &\quad + a_r \cdot \underline{g}_r^0 \cdot H_r^0 \cdot H_r^0 \cdot \underline{g}_r^0 \} \\ &= 0 \text{ at } a_r = a_r^*; \quad r = 1, 2, \dots, K \quad \dots (5.17) \end{aligned}$$

These equations are linear in the unknowns a_r^* , and all the $H_{\underline{g}}$ terms needed are supplied by K new OLF solutions after exploratory steps, the same number as for $H \cdot \hat{\underline{g}}_c$ in the previous methods. Separate step-lengths for each time interval quicken the minimisation at the expense of inverting a $K * K$ matrix at each step and, perhaps more

importantly, obtaining schedules which do not obey the discharge constraints until the hill-climber has converged.

5.3.2 Second-Derivative Hill-Climber

It appears attractive to find the optional discharge rates in one step by Newton's method. If

$$f_k = \frac{1}{2} \underline{u}_k^T H_k \underline{u}_k + b_k^T \underline{u}_k + c_k \quad \dots \quad (5.18)$$

then substituting in (5.3), solving for \underline{u}_k , summing over K and inserting the discharge constraints gives

$$\underline{u}_k = -H_k^{-1} (b_k + \underline{\lambda}) \quad \dots \quad (5.19)$$

where

$$\underline{\lambda} = -\left[\sum_{k=1}^K H_k^{-1} \right]^{-1} \left(\underline{q} + \sum_{k=1}^K \{H_k^{-1} b_k\} \right) \quad \dots \quad (5.20)$$

As they stand, these expressions involve the inversion of $K + 1$ matrices of the same dimensions as the number of discharge rates N and $K + 2$ matrix-vector products. However, the off-diagonal terms of the H_k 's are often small. Replacing each H_k by its principal diagonal makes the matrix inversions trivial and the second derivatives then require only one extra OLF per time interval. The expressions simplify to

$$\lambda_i = -\left(q_i + \sum_{k=1}^K \frac{b_{ik}}{h_{iik}} \right) / \sum_{k=1}^K \frac{1}{h_{iik}} \quad i = 1, 2, \dots, N \quad \dots \quad (5.21)$$

and

$$u_{ik} = -(b_{ik} + \lambda_i) / h_{ik} \quad \begin{matrix} i = 1, 2, \dots, N; \\ k = 1, 2, \dots, K \end{matrix} \quad \dots (5.22)$$

Similarly, we can minimise $\underline{g}_c^T \cdot \underline{g}_c$ in place of f by setting

$$u_{ik} = -\frac{\lambda_i}{2h_{ik}} - \frac{b_{ik}}{h_{ik}} \quad \begin{matrix} i = 1, 2, \dots, N; \\ k = 1, 2, \dots, K \end{matrix} \quad \dots (5.23)$$

where

$$\lambda_i = -2(q_i + \sum_{k=1}^K \frac{b_{ik}}{h_{ik}}) / \sum_{k=1}^K \frac{1}{h_{ik}} \quad i = 1, 2, \dots, N \quad \dots (5.24)$$

5.4 MONTE-CARLO TESTS OF HILL-CLIMBER PERFORMANCE

Hill-climber performance was tested by Monte-Carlo simulations, each of sixteen 4-step runs to refine six-hour schedules for four hydro plants. Deviations of the cost vs discharge-rate relations from quadratic form were simulated by perturbing every exploratory cost and slope evaluation so that $\hat{f}_0 - f_0$ and $\hat{g}_c^0 - g_c^0$ were uniformly distributed between $1 \pm R$ times their correct values. The accuracy and robustness of six hill-climbers designated A to F corresponding to equations (5.8), (5.10), (5.13), (5.14), (5.22) and (5.23) respectively, were compared for a range of R . The starting schedule used generation at the hydro plant with largest specified discharge to take up all the demand variation.

Fig. 5.3 and Table A2.1 show results for perfectly quadratic costs, while Fig. 5.4 and Table A2.2 summarise the costs achieved with R varied over the range 0.1 to 1.2.

For moderate perturbation, gradient methods A and C perform well, while B and D do not. On examining the expressions for a^* , we see that B and D utilise the small difference $\hat{f}^0 - f^0 + \hat{a} \underline{g}_c^0 T \underline{g}_c^0$ of relatively large numbers, and perturbation of \hat{f}^0 can have a disastrous effect for some step sizes \hat{a} . In A and C, conditioning is much better and unaffected by \hat{a} , virtually identical results being obtained over several decades of \hat{a} (Table A2.3). Method A is rather better than C for R below about 0.4, but above 0.4 the positions are reversed. Careful choice of \hat{a} gives good results with B and D, but these methods were rejected as too vulnerable. For these tests $\hat{a} \approx 1.0$ gave adequate results.

For R below about 0.4 and 0.6 respectively, the quadratic methods E and F remain far superior to A and C, but large perturbations cause them to break down well before the gradient techniques, misbehaving badly at $R = 0.8$. Method C performs virtually as well as $R = 1.2$ as at low R and still converges at $R = 3.0$.

To check the effects of diagonal dominance of the cost Hessians, the cross-product terms of each $f_k(\underline{u}_k)$ were halved, adjusting the linear terms to keep the same schedules optimal. The gradient methods are almost unaffected by the change. The second order methods improve, with method E now performing almost as well as method F until the onset of divergence below $R = 0.8$. These results are summarised in Fig. 5.5 and Table A2.4.

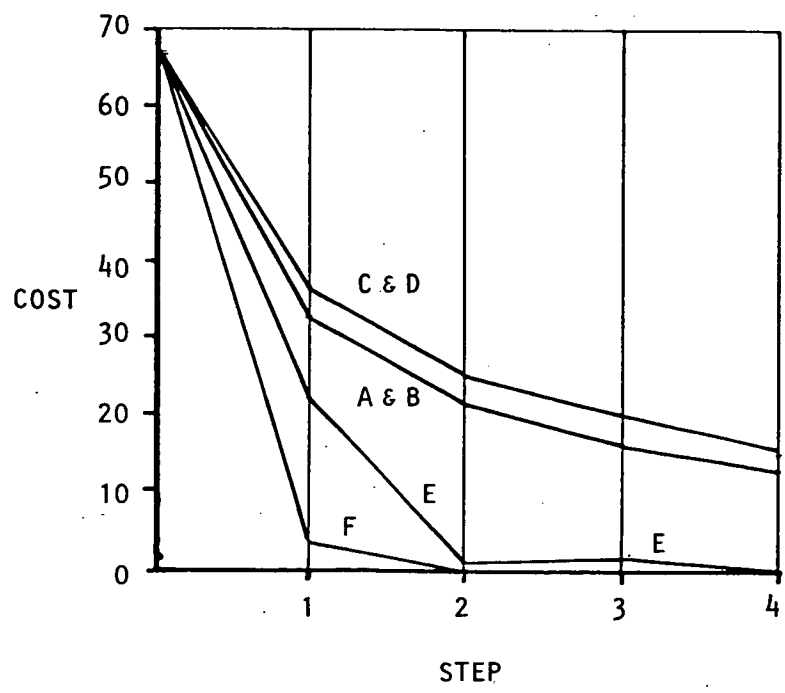


Fig 5.3 Performance of hill climbers on perfectly quadratic hills

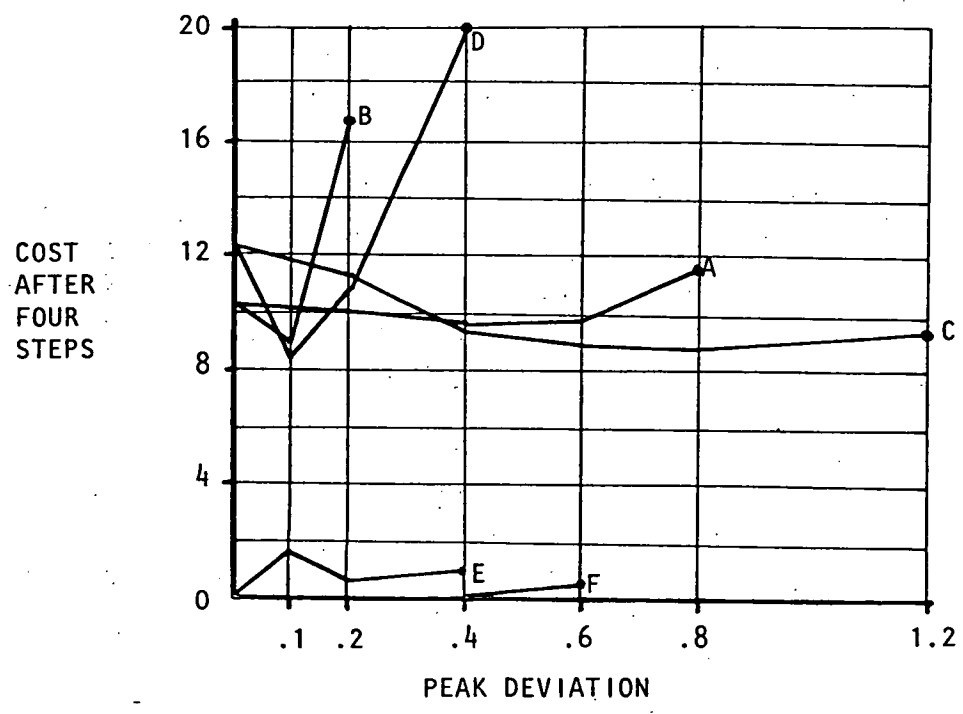


Fig 5.4 Performance of hill climbers with a randomly perturbed quadratic hill

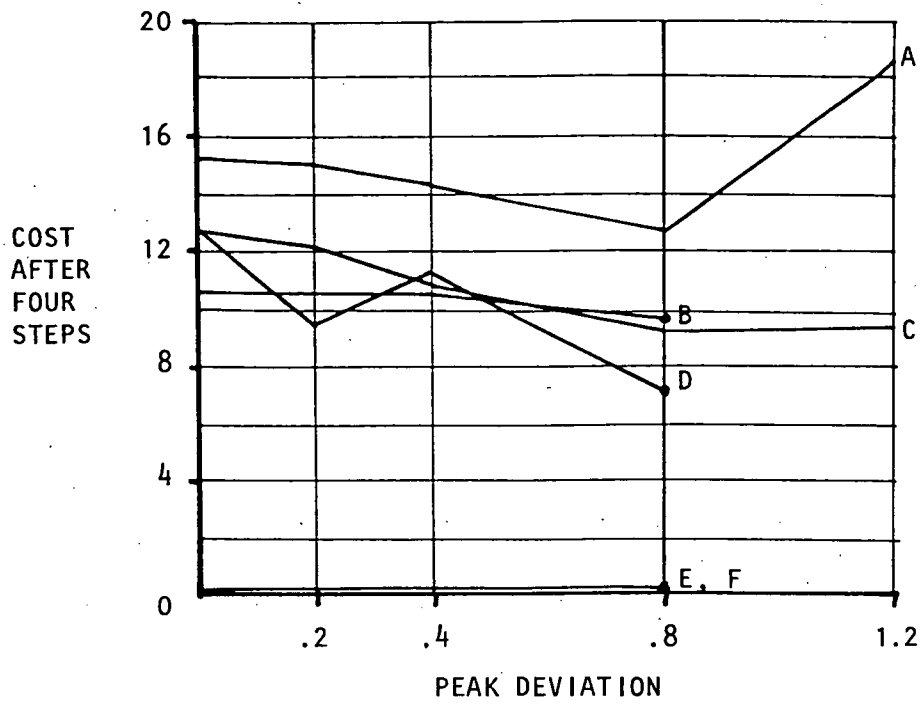


Fig 5.5 Performance of hill climbers with a randomly perturbed quadratic hill. Off diagonal terms halved

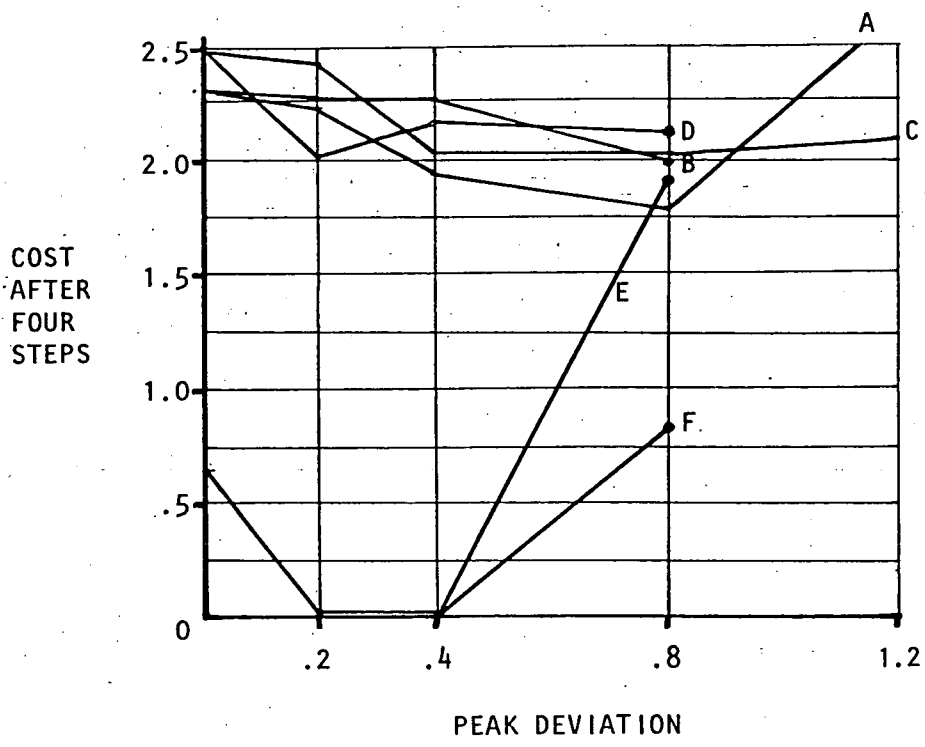


Fig 5.6 Performance of hill climbers with a randomly perturbed quadratic hill. Step loading starting point

As a final check on performance, methods A - F were tested with a step-loaded search starting point in place of the constant generation starting point used in other tests. These results are summarised in Fig. 5.6 and Table A2.5. All methods exhibited greater stability and faster convergence from this more realistic starting point.

5.5 SUMMARY

The hydrothermal power system may, for scheduling purposes, be decomposed into an electrothermal subsystem solved separately in each time interval, and a group of hydro subsystems, one for each river system as illustrated by Fig. 5.2.

Our technique involves refinement of an initial step-loading schedule which sets unit commitment, employing optimal load flow computations to schedule the electrothermal subsystem and enforce electrothermal subsystem constraints, alternating with a hill-climber using the OLF results to improve the hydro schedule. The hill-climber is chosen for its performance over very few steps.

Four gradient and two second-derivative hill-climbing algorithms were subjected to Monte-Carlo tests on simulated scheduling of a four-plant hydro-power system and assessed on the basis of their performance after up to four steps. Of the methods tested, two of the gradient methods proved very robust and immune to poorly chosen step length, method C using expression (5.8) for its step-length being superior to method A for large perturbations from a quadratic Hessian. The second order methods were much faster than the gradient methods for small perturbations from a quadratic

Hessian, with method F using expression (5.23) being the more robust.

The choice between gradient method C and second-order method F is determined by the scheduling problems being considered. The shape of the objective function will be investigated more closely in Chapters 8 and 9.

5.6 REFERENCES

- [1] ADBY, P.R. and DEMPSTER, M.A.H. (1974) Introduction to optimisation methods. London, Chapman and Hill.

- [2] BERNHOLTZ, B. and GRAHAM, L.J. (1960-61) Hydrothermal economic scheduling Part I AIEE Trans. Power Appar. Sys. Vol. 79 pp. 921-932. Parts II-IV AIEE Trans. Power Appar. Sys. Vol. 80 pp. 1089-1107.

- [3] BONAERT, A.P., EL-ABIAD, A.H. and KOIVO, A.J. (1971) On the optimal scheduling of hydrothermal power systems. Research report TR-EE 70-21, School of Electrical Engineering, Purdue University.

- [4] CARPENTIER, J. and SIRIOUX, J. (1963) L'optimisation de la production à electricité de France. Bull de la Société Francaise des Electriciens. March pp. 121-129.

- [5] DILLON, T.S. (1974) Problems of optimal economic operation and control of integrated-(hydrothermal) and thermal power systems. Ph. D. Thesis, Department of Electrical Engineering Monash University.

- [6] DILLON, T.S. and MORSZTYN, K. (1969) Optimal control of integrated power systems. Research report MEE 69-3, Department of Electrical Engineering, Monash University.

- [7] DILLON, T.S. and MORSZTYN, K. (1972) A new theoretical and computational approach to the exact solution of the problem of optimal control of integrated (hydrothermal) power systems. 5th IFAC World Congress, Paris, June.

- [8] FUKAO, T., YAMAZAKI, T., KIMURA, M. (1959) An application of dynamic programming to economic operation of a power system. Electrotechnical Journal of Japan. Vol. 5 No. 2 pp. 64-68

- [9] KIRCHMAYER, L.K. (1959) Economic control of interconnected systems. John Wiley and Sons.

- [10] MANTERA, I.G.M. (1971) Optimum hydro-electric power generation scheduling by analogue computer. Proc IEEE. Vol. 118 pp. 110-116.

- [11] TINNEY, W.F. and HART, C.E. (1967) Power flow solution by Newton's method. Trans IEEE Pwr. Appar. sys. Vol. 86 pp. 1449-1460.

- [12] VAN NESS, J.E. and GRIFFIN, J.H. (1961) Elimination methods for load flow studies. Trans AIEE Pwr. Appar. Sys. Vol. 80 pp. 299-304.

6. COMPUTATIONAL METHODS FOR SCHEDULING THE ELECTROTHERMAL SUBSYSTEM

As mentioned in Chapter 1, the short-term scheduling of the electrothermal subsystem is a static optimisation problem. Such problems may be approached by direct or indirect methods. The direct approach generates a sequence of feasible solutions such that each solution is no worse, and is usually better than the preceding one. This sequence is terminated when further improvement in the solution is unlikely to justify the extra computation involved. The simplex method in linear programming and various gradient techniques are examples of this approach. In the indirect approach, assuming continuity, continuous differentiability and convexity, the original problem is replaced by the transformed problem of finding the set of solutions satisfying a set of equations, usually the first-order Kuhn-Tucker necessary conditions, characterising the optimum. In this approach, not all independent variables are identifiable beforehand. For example, if constraint $P_{Gi} - \hat{P}_{Gi} \leq 0$ is inactive, then P_{Gi} is independent, and the associated dual variable is zero, but if this constraint is active, that is, $P_{Gi} = \hat{P}_{Gi}$, then P_{Gi} is fixed and the associated dual variable becomes an independent variable. The complexity of computational logic required to keep track of the independent variables is a disadvantage of this method.

The direct methods, while lacking the mathematical elegance of the indirect methods, have the advantage that they may be cut short at any step to yield an improved feasible solution if computing

time is limited. Owing to the complexity of the power system scheduling problem, most indirect methods resort to an iterative technique starting from an initial point not necessarily satisfying all constraints, and progressing to a final solution satisfying all constraints within the desired tolerance.

In power system scheduling, the indirect methods often circumvent problems associated with the direct methods, such as finding first and higher order derivatives of the objective function with respect to control variables, and the convergence difficulties of search techniques not using these. This is possibly at the expense of introducing similar problems in satisfying boundary values.

The indirect methods of scheduling the electrothermal subsystem which are summarised in the following sections are those of Carpentier and Sirieux [4, 5], Dommel and Tinney [11], Rashmed and Kelly [24], and Peschon et al. [23], while the direct methods are represented by those of Dauphin, Feingold and Spohn [6], Nabona and Freris [22], Smith and Tong [30], Baumann [2], and Dopazo et al. [12]. Of these, references [2], [23], [30] deal only with the problem of reactive generation scheduling.

Two mutually related methods not easily fitting the framework of direct and indirect methods are also described. These are the Sequential Unconstrained Minimisations Technique [7, 8, 10, 25, 27], and the Sequential Linearly Constrained Minimisations Technique [7, 9] relying on transforming the problem into a sequence of unconstrained or linearly constrained minimisations with penalty functions attaching the constraints to the objective function.

6.1 THE METHOD OF DAUPHIN, FEINGOLD AND SPOHN [6]

A transformed problem is defined by linearising the load-flow equations, hence the nonlinear optimisation is replaced by a sequence of linear programming problems which may then be solved by one of the many powerful techniques available. The cost function for thermal generation must be linear

$$F(P_{Gi}) = a_{0i} + a_{1i} P_{Gi} \quad i = 1, 2, \dots, \ell \quad \dots (6.1)$$

The function to be minimised is

$$F_T = \sum_{i=1}^{\ell} a_{1i} P_{Gi} \quad \dots (6.2)$$

If there is a change ΔP_G in the value of P_G , then a change of

$$\Delta F_T = \sum_{i=1}^{\ell} a_{1i} \Delta P_{Gi} \quad \dots (6.3)$$

in total cost results. If $\Delta F_T = 0$, then P_G was not a minimum.

The linearised system load-flow equations are

$$\begin{bmatrix} \frac{\partial I}{\partial \phi} & \frac{\partial I}{\partial V} \\ \frac{\partial K}{\partial \phi} & \frac{\partial K}{\partial V} \end{bmatrix} \begin{bmatrix} \Delta \phi \\ \Delta V \end{bmatrix} = \begin{bmatrix} \Delta P_G \\ \Delta Q_G \end{bmatrix} \quad \dots (6.4)$$

and the compatibility relation is

$$\mathcal{F}_{T1} = \sum_{i=1}^N (\beta_i \Delta P_{Gi} + \gamma_i \Delta Q_{Gi}) = 0 \quad \dots (6.5)$$

$$\mathcal{F}_{T2k} = \sum_{i=k}^N \beta_k \left(\Delta P_{Gi} - \left(\frac{\partial K}{\partial \phi} \right)_k \cdot \Delta V_k \right) + \sum_{\substack{i=1 \\ i \neq k}}^N \gamma_i^k \left(\Delta Q_{Gi} - \left(\frac{\partial K}{\partial V_k} \right) \cdot \Delta V_k \right) = 0$$

$$k = 1, 2, \dots, N \quad \dots (6.6)$$

where k is the row and column eliminated in the solution at each stage.

I.e. minimise N

$$F_T = \sum_{i=1}^N a_{1i} \cdot \Delta P_{Gi} - \mathcal{F}_{T1} - \sum_{k=1}^N \mathcal{F}_{T2k} \quad \dots (6.7)$$

subject to $\mathcal{F}_{T1} = 0$

$$\mathcal{F}_{T2k} = 0; \quad k = 1, 2, \dots, N \quad \dots (6.8)$$

$$\left. \begin{aligned} \tilde{P}_{Gi} - P_{Gi} &\leq \Delta P_{Gi} \leq \hat{P}_{Gi} - P_{Gi} \quad \dots (a) \\ \tilde{Q}_{Gi} - Q_{Gi} &\leq \Delta Q_{Gi} \leq \hat{Q}_{Gi} - Q_{Gi} \quad \dots (b) \\ \tilde{V}_i - V_i &\leq \Delta V_i \leq \hat{V}_i - V_i \quad \dots (c) \end{aligned} \right\} \quad \dots (6.9)$$

and for load buses $\Delta P_{Gi} = \Delta Q_{Gi} = 0 \quad \dots (6.10)$

Corrections ΔP_{Gi} , ΔQ_{Gi} , ΔV_i are applied and a new linearisation is performed around the new operating point. Equations (6.3) to (6.7) are linear in ΔP_G , ΔQ_G and ΔV , so the solution algorithm is:

1. Solve a load flow for an initial operating point and find the cost $F_T(P_G)$.
2. Compute $\beta, \gamma, \beta_k, \gamma_k$.
3. Solve the linear programming problem for $\Delta P_G, \Delta Q_G$ and ΔV . Terminate if these are sufficiently small.
4. Apply corrections

$$P_{Gi}^{\ell+1} = P_{Gi}^{\ell} + c \cdot \Delta P_{Gi}$$

$$Q_{Gi}^{\ell+1} = Q_{Gi}^{\ell} + c \cdot \Delta Q_{Gi}$$

$$V_i^{\ell+1} = V_i^{\ell} + c \cdot \Delta V_i$$
 at all buses except the slack bus.
 $c = \text{step size} = 1$ initially.
5. Solve the load flow for $Q_G, V, P_{\text{slack}}, Q_{\text{slack}}$ and check the constraints.
6. Compute the new cost $F_T(P_G)$.
7. If $F_T^{\ell+1} < F_T^{\ell}$, then go to step 2 otherwise reduce c and go to step 4.

This technique has the advantage that it extends linear programming techniques to the scheduling problem, and so has no convergence difficulties within each subproblem, although the convergence characteristics of the overall nonlinear problem are unknown. It may be possible to employ the same linearisation for more than one step, so economising on computing. The method requires a linear cost characteristic, so it is not well suited to systems with nozzle-governed thermal plant (see section 3.4). A major advantage of this method is the feasibility of each intermediate solution.

As noted by Dillon [7], this method has strong similarity

to the Method of Approximation Programming (MAP) derived by Griffith and Stewart [16] for a general nonlinear programming problem, and should have similar convergence properties.

It was found [15] that during solution, MAP may oscillate between the vertices of the feasible region, however, this can usually be inhibited by reducing the allowable range of ΔP_{Gi} .

6.2 THE METHOD OF NABONA AND FRERIS [22]

Expressing each constraint as a linear relation, Nabona and Freris use quadratic programming to arrive at minimum cost with respect to changes in all variables within permissible limits. The method is implemented as an integral part of a Newton-Raphson load-flow algorithm.

The cost of generating sets is assumed able to be approximated by a quadratic between minimum and maximum power limits. The cost of generation becomes

$$\$ = C_b + \underline{C}_\ell^T \cdot \underline{P}_g + \underline{P}_g^T \cdot [C_q] \cdot \underline{P}_g \quad \dots \quad (6.11)$$

where C_b is the basic cost, \underline{C}_ℓ is the N-vector of linear cost coefficients and $[C_q]$ is the diagonal $N \times N$ matrix of quadratic cost co-efficients.

Denoting the vector of control variables by \underline{u} , a change $\Delta \underline{u}$ results in a change in cost

$$\Delta \$ = (\underline{C}_\ell^T + 2\underline{P}_g^T \cdot [C_q]) \cdot \Delta \underline{P}_g + \Delta \underline{P}_g^T \cdot [C_q] \cdot \Delta \underline{P}_g \quad \dots \quad (6.12)$$

It is possible, through the sensitivity relations for \underline{P}_g to express $\Delta \$$ in terms of the changes $\Delta \underline{u}$ in the vector of control variables.

If

$$\Delta P_g \approx \left[\frac{\partial P_g}{\partial u} \right] \Delta u \quad \dots (6.13)$$

where $\left[\frac{\partial P_g}{\partial u} \right]$ is the matrix of sensitivity co-efficients for power generation, then equation (6.12) can be rewritten as

$$\Delta \mathcal{L} = (C_l^T + 2 P_g^T \cdot [C_q]) \cdot \left[\frac{\partial P_g}{\partial u} \right] \Delta u + \Delta u^T \cdot \left[\frac{P_g}{\partial u} \right]^T \cdot [C_q] \cdot \left[\frac{\partial P_g}{\partial u} \right] \Delta u \quad \dots (6.14)$$

The quadratic cost function of equation (6.14) can be minimised, subject to constraints, in a finite number of steps by quadratic programming. If the sensitivity relations and cost curve are sufficiently good approximations, the algorithm will achieve optimal economic dispatch in one iteration. Practical tests using Beale's quadratic programming technique [3] confirmed this. It must be noted that one iteration of a quadratic programming algorithm entails one cycle of the control variables, with one load-flow solution for variation of each variable.

Nabona and Freris reduced the amount of computing necessary by re-using the optimally ordered and factored Jacobian of the load flow equations computed in the base-case load flow solution. Linear programming was also applied to this problem as an alternative to quadratic programming, but convergence was not achieved. The problem of minimum reactive generation was also considered.

6.3 THE METHOD OF CARPENTIER AND SIRIOUX [4, 5]

The general problem of minimising instantaneous operating costs in a power system subject to equality constraints imposed by the physical laws governing the transmission system, that is, the load-flow equations, as well as the inequality constraints imposed by the equipment ratings, was first formulated in 1961 by Carpentier [4] of Electricité de France. The resulting problem was shown to be one of nonlinear programming. The Kuhn-Tucker first-order necessary conditions were applied to derive a set of equations satisfied at the optimum. Carpentier treated as a particular case the optimisation of real power assuming that the network had a known unchanging voltage profile. A successful computer programme was developed for solving the optimisation equations one by one, as in the Gauss-Seidel method, for a 50-node system obeying this assumption.

Siriaux [5], also of Electricité de France, formulated the optimisation problem in an almost identical manner, arriving at the Kuhn-Tucker necessary conditions by engineering intuition based on Kirchmayer's concept of marginal costs [17, 18, 19].

In the Carpentier-Siriaux formulation, the cost function for an N-node thermal power system is

$$F_T = f(P_{Gi}, Q_{Gi}); \quad i = 1, 2, \dots, N \quad \dots \quad (6.15)$$

but in a modern system $\frac{\partial F}{\partial P_{Gi}} \gg \frac{\partial F}{\partial Q_{Gi}}$, so

$$F_T \approx f(P_{Gi}) \quad \dots \quad (6.16)$$

The equations of injection are

$$P_{Ti} - P_{Gi} + P_{Di} = 0 \quad \dots (6.17)$$

$$Q_{Ti} - Q_{Gi} + Q_{Di} = 0 \quad \dots (6.18)$$

with system constraints

$$P_{Gi}^2 + Q_{Gi}^2 < \hat{S}_{Gi}^2 \quad \dots (a) \quad \check{P}_{Gi} < P_{Gi} \quad \dots (b)$$

$$Q_{Gi} < \hat{Q}_{Gi} \quad \dots (c) \quad \check{Q}_{Gi} < Q_{Gi} \quad \dots (d)$$

$$V_i < \hat{V}_i \quad \dots (e) \quad \check{V}_i < V_i \quad \dots (f)$$

and stability constraint

$$|\phi_i - \phi_a| < \hat{T}_{ia} \quad \dots (g) \quad \dots (6.19)$$

The problem comprises minimisation of F_T with respect to variables P_{Gi} , Q_{Gi} , V_i and ϕ_i while satisfying constraints (6.16) - (6.19).

Classical Lagrange Multiplier theory is inadequate when dealing with inequality constraints. The Kuhn-Tucker theorem states that given cost, $F(\underline{x})$, of vector variable \underline{x} , inequality constraints

$$G(\underline{x}) \leq 0 \quad \dots (6.20)$$

equality constraints

$$H(\underline{x}) = 0 \quad \dots (6.21)$$

and assuming convexity for F , G and H , then $F(\underline{x})$ is a minimum if all

$$\frac{\partial L}{\partial \underline{x}} = 0 \text{ with}$$

$$L = F(\underline{x}) + \underline{\alpha}^T \cdot G(\underline{x}) + \underline{\beta}^T \cdot H(\underline{x}) \quad \dots (6.22)$$

and

$$\alpha_i \cdot G_i(\underline{x}) = 0 \quad \dots (6.23)$$

The α_i and β_i are dual variables associated with inequality and

equality constraints (6.20) and (6.21) respectively. Condition (6.23) implies that $\alpha_i = 0$ if constraint $G_i(\underline{x}) \leq 0$ is inactive and $\alpha_i > 0$ when it is active.

The generalised Lagrangian for a thermal power system can be written as

$$\begin{aligned}
 L = & \sum_{i=1}^N F_{Ti}(P_{Gi}) + \sum_{i=1}^N \lambda_i [P_{Ti}(V_i, \phi_i) - P_{Gi} + P_{Di}] + \sum_{i=1}^N \mu_i [Q_{Ti}(V_i, \phi_i) - Q_{Gi} + Q_{Di}] \\
 & + \sum_{i=1}^N M_i (P_{Gi}^2 + Q_{Gi}^2 - \hat{S}_{Gi}^2) + \sum_{i=1}^N m_i (\tilde{P}_{Gi} - P_{Gi}) + \sum_{i=1}^N e_i (Q_{Gi} - \hat{Q}_{Gi}) + \sum_{i=1}^N e_i^1 (\tilde{Q}_{Gi} - Q_{Gi}) \\
 & + \sum_{i=1}^N u_i (V_i - \hat{V}_i) + \sum_{i=1}^N u_i^1 (\tilde{V}_i - V_i) + \sum_{i=1}^N \sum_{\substack{a=1 \\ a \neq i}}^N t_{ia} (\phi_i - \phi_a - \hat{T}_{ia}) \\
 & \dots \quad (6.23a)
 \end{aligned}$$

At the optimum, differentiating with respect to P_{Gi} and Q_{Gi}

$$\frac{\partial L}{\partial P_{Gi}} = 0 \Rightarrow \lambda_i = \frac{\partial F_{Ti}}{\partial P_{Gi}} + 2M_i P_{Gi} - m_i \quad \dots \quad (6.24)$$

$$\frac{\partial L}{\partial Q_{Gi}} = 0 \Rightarrow \mu_i = 2M_i Q_{Gi} + e_i - e_i^1 \quad \dots \quad (6.25)$$

and differentiation with respect to V_i and ϕ_i

$$\frac{\partial L}{\partial \phi_i} = 0 \rightarrow \lambda_i \frac{\partial P_{Ti}}{\partial \phi_i} + \sum_a \lambda_a \frac{\partial P_{Ta}}{\partial \phi_i} + \mu_i \frac{\partial Q_{Ti}}{\partial \phi_i} + \sum_a \mu_a \frac{\partial Q_{Ta}}{\partial \phi_i} + \sum_a (t_{ai} - t_{ia}) = 0$$

... (6.26)

$$\frac{\partial L}{\partial V_i} = 0 \rightarrow \lambda_i \frac{\partial P_{Ti}}{\partial V_i} + \sum_a \lambda_a \frac{\partial P_{Ta}}{\partial V_i} + \mu_i \frac{\partial Q_{Ti}}{\partial V_i} + \sum_a \mu_a \frac{\partial Q_{Ta}}{\partial V_i} + u_i - u_i^1 = 0$$

... (6.27)

In addition, the power flow equations (6.17) and (6.18) and the exclusion equations:

$$M_i (P_{Gi}^2 + Q_{Gi}^2 - \hat{S}_{Gi}^2) = 0 \quad \dots \quad (a)$$

$$m_i (\check{P}_{Gi} - P_{Gi}) = 0 \quad \dots \quad (b)$$

$$e_i (Q_{Gi} - \hat{Q}_{Gi}) = 0 \quad \dots \quad (c)$$

$$e_i^1 (\check{Q}_{Gi} - Q_{Gi}) = 0 \quad \dots \quad (d)$$

$$u_i (V_i - \hat{V}_i) = 0 \quad \dots \quad (e)$$

$$u_i^1 (\check{V}_i - V_i) = 0 \quad \dots \quad (f)$$

$$t_{ia} (\phi_i - \phi_a - \hat{T}_{ia}) = 0 \quad \dots \quad (g) \quad \dots \quad (6.28)$$

must be satisfied at the optimum. If $P_{Gi} = Q_{Gi} = 0$ at a node, then the corresponding equations (6.26) and (6.27) disappear.

The variables $\lambda_i, \mu_i, M_i, m_i, e_i, e_i^1, u_i, u_i^1, t_{ia}$ are the dual variables of Kuhn and Tucker. Carpentier [4] showed that at the optimum, λ_i and μ_i measure the incremental cost of active and reactive consumption at node i . The other dual variables measure marginal costs associated with constraints (6.19 a-g) respectively.

To overcome the previously mentioned difficulties arising from independent variables not all being identifiable beforehand, Carpentier [4, 5] proposed the following computational scheme:

1. A set of non-constrained primary variables P, Q, V and ϕ is assumed which initially need not satisfy the power flow equations (6.17) and (6.18). A set of dual variables λ and μ is also assumed.
2. This set is substituted into one optimisation equation, and the change in one of the variables required to satisfy this equation is computed.
3. If the required change is allowed by the constraint equations, it is executed; if not, then all or part of the required change is forced into another variable, and the dual variable associated with the constrained primary variable is introduced into the appropriate optimisation equation.
4. Proceeding to the next equation, the corrected value obtained in step 3 is used. A permissible change for another variable is computed.

The choice of control variable for each equation is usually made on a physical basis; for example, if node i is a production node, then P_{Gi} is the usual control variable for the active power flow equation, while if it is a consumption node, then the voltage phase angle ϕ_i is the usual choice.

Because of the logic involved, implementation of this scheme for a large power system is complex. Due to its step-by-step approach, the method has doubtful convergence properties [11] particularly when load buses with particularly high or particularly low power factors are present.

6.4 THE METHOD OF DOMMEL AND TINNEY [11] AND RELATED METHODS [23, 24]

Developed at Bonneville Power Authority (BPA), Dommel and Tinney's method is built around their very fast load-flow programmes [28, 31]. As noted by Dommel and Tinney, a similar method was also developed in the USSR [13, 20, 29]. We can write the load-flow equations (3.5) and (3.6) in control system notation as a $2n$ - vector equation

$$\underline{g}(\underline{x}, \underline{u}, \underline{p}) = \underline{0} \quad \dots \quad (6.29)$$

where \underline{u} is the vector of independent (control) variables,

\underline{x} is the vector of dependent (state) variables, and

\underline{p} is the vector of predetermined parameters including real and VAR demands.

Beginning from an initial estimated control vector \underline{u}^0 , a new \underline{u} giving improved performance is chosen using a gradient technique, and is used as input data to the load-flow algorithm to

provide a new set of state variables. The process is complicated by constraints on both control and state variables. Control constraints are accommodated by setting a variable equal to its limit when a gradient step would cause it to exceed this value. Constraints on state variables are functional constraints, and are handled by external penalty functions, defining a transformed problem such that

$$F' = F(\underline{x}, \underline{u}, \underline{p}) + \sum r_i (x_i - \hat{x}_i)^2 + \sum r_i' (\check{x}_i - x_i)^2 \quad \dots (6.30)$$

with only violated constraints included in F' .

Incorporating both types of constraints, the object is to minimise the unconstrained Lagrangian function

$$L = F'(\underline{x}, \underline{u}, \underline{p}) + \underline{\lambda}^T \cdot \underline{g}(\underline{x}, \underline{u}, \underline{p}) + \underline{\hat{\mu}}^T \cdot (\underline{u} - \underline{\hat{u}}) + \underline{\check{\mu}}^T (\underline{\check{u}} - \underline{u}) \quad \dots (6.31)$$

where $\underline{\hat{\mu}}$ and $\underline{\check{\mu}}$ are the vectors of dual variables associated with the upper and lower limits on control variables \underline{u} and $\underline{\lambda}$ is the vector of dual variables associated with the load flow equations. The necessary conditions for a minimum are

$$\begin{aligned} \underline{\nabla}_{\underline{x}} L &= \frac{\partial F'}{\partial \underline{x}} + \left[\frac{\partial \underline{g}}{\partial \underline{x}} \right]^T \cdot \underline{\lambda} = \underline{0} \\ \underline{\nabla}_{\underline{u}} L &= \frac{\partial F'}{\partial \underline{u}} + \left[\frac{\partial \underline{g}}{\partial \underline{u}} \right]^T \cdot \underline{\lambda} + \underline{\hat{\mu}} = \underline{0} \\ \underline{\nabla}_{\underline{\lambda}} L &= \underline{g}(\underline{x}, \underline{u}, \underline{p}) = \underline{0} \\ \underline{\nabla}_{\underline{\hat{\mu}}} L &= \underline{\hat{\mu}} - \underline{\hat{\mu}} = \underline{0} ; \underline{\nabla}_{\underline{\check{\mu}}} L = \underline{\check{\mu}} - \underline{\check{\mu}} = \underline{0} \quad \dots (6.32) \end{aligned}$$

where $\mu_i = \hat{\mu}_i$ if $\mu_i > 0$
 $\mu_i = -\check{\mu}_i$ if $\mu_i < 0$

Since in this scheme, a control variable on reaching its limit becomes a dependent variable, releasing another variable as an independent variable, $\check{\underline{u}}$ and $\hat{\underline{u}}$ must be zero at all times, simplifying the necessary conditions.

The algorithm is

Step 1: Select an initial control vector $\underline{u}^{(0)}$.

Step 2: Find a feasible power flow by Newton's Method.

Step 3: Solve the linear equation (6.32) for $\underline{\lambda}$. Note that $\begin{bmatrix} \partial g \\ \partial x \end{bmatrix}$ is the Jacobian of the power flow

equations, for which the factored inverse is available from Step 2.

Step 4: Compute the gradient $\nabla_{\underline{u}} f = \frac{\partial L}{\partial \underline{u}}$ and find a new

control vector using the gradient method

$$\underline{u}^{(\ell+1)} = \underline{u}^{(\ell)} - c \cdot \nabla_{\underline{u}} f \quad \dots \quad (6.33)$$

where c is the step-length constant. If

$u_i^{(\ell+1)}$ exceeds its limit, it is set equal to that limit.

Step 5: If $\nabla_{\underline{u}} f$ is sufficiently small, the minimum has been reached, otherwise return to Step 2.

Dommel and Tinney [11] tested four gradient adjustment algorithms: A second-order method using a diagonalised Hessian

matrix, an optimal gradient method, the method of parallel tangents (PARTAN) and a mixed method which combined the optimal gradient method and the diagonalised second order method, changing from the former to the latter when at least one gradient component changed sign. The use of a simple steepest descent formula is critically dependent on correct choice of c , resulting in slow progress if c is too small, and oscillatory behaviour on valleys and ravines in the cost surface if c is too large. The mixed method was found to perform best [11], the second order method having problems with control interaction and PARTAN performing well only when there were few control parameters and cost contours were relatively undisturbed by the introduction of penalty functions.

No systematic method for choice of the penalty coefficients r_i was given. Although improper choice of these results in either successive violation of constraints or severe distortion of the cost surface, the penalty coefficients can usually be ascertained after some experience with the method.

This algorithm was preceded by one from Peschon et. al. [23] for reactive VA optimisation. In this, penalty functions were not used to apply constraints on state variables \underline{x} , the variable interchange suggested by Carpentier [4] (Section 6.3) being used instead. Rather than using the gradient method of Dommel and Tinney, Peschon et. al. used the method of regular falsi (false position) to compute the improved control vector.

The Dommel and Tinney method is very similar to the reduced gradient methods [1, 16]. Peschon et. al. [23] have applied the

generalised reduced gradient algorithm developed by Abadie [1] to the optimal load flow computation, circumventing the difficulty of constraints on state variables in the following manner. Whenever a state variable x_i reaches its limit \check{x}_i or \hat{x}_i , the problem variables are relabelled. The formerly dependent variable x_i now becomes an independent variable u_j and a formerly independent variable becomes a dependent variable. The new dependent variable is chosen so that it is free to move, at least during the next iteration. The new independent variable is simply held at the limit it reached, at least for one iteration. This formalises the method suggested by Carpentier [4] (Section 6.3) for handling constraints.

Dillon [7] notes similarities in handling of equality constraints between the B.P.A. method and the sequential gradient restoration method of Miele, Huang and Heidemann [21]. In both of these methods, there is a gradient phase in which L is reduced, followed by a restoration phase when equality constraints are enforced. While the step taken in the gradient phase is the same in both methods, the restoration phase differs, Miele et. al. using a Gauss type step of the form

$$\underline{\Delta x} = -\underline{g}_x(\underline{x}, \underline{u}) \cdot [\underline{g}_x^T(\underline{x}, \underline{u}) \cdot \underline{g}_x(\underline{x}, \underline{u})]^{-1} \underline{g} \dots \quad (6.34)$$

in place of the Newton-Raphson method used by B.P.A.

An alternative way of handling constraints on state variables is suggested by Dillon [7]. The constraints on the dependent variables are of the form

$$\check{x}_i \leq x_i \leq \hat{x}_i \quad \dots \quad (6.35)$$

If new variables v_i are defined by

$$x_i = x_i + (\hat{x}_i - \check{x}_i) \cdot \sin^2 v_i \quad \dots \quad (6.36)$$

then variables v_i take on such values as ensure that constraints on the state variables x_i are fulfilled. If these new variables v_i are treated as the dependent variables, penalty functions may be removed and the method remains valid. The transformation is such that it does not affect the sparsity of the Jacobian matrix in the Newton-Raphson solution. It does, however, affect any near-linearity which may be present.

In keeping with the notion of physical or "hard" constraints and imposed or "soft" constraints (Section 3.3.6), the most effective method of handling them appears to be a hybrid of the generalised reduced-gradient method to handle physical constraints and a penalty function method handling imposed constraints.

Based on the Dommel and Tinney formulation, Rashmed and Kelly [24] computed a correction vector

$$\underline{\Delta u} = -[H]^{-1} \cdot \underline{\nabla_u f} \quad \dots \quad (6.37)$$

where H is the Hessian matrix of second-order partial derivatives of the Lagrangian function, L , with respect to control variables \underline{u} . It is in general extremely sparse, but this may not always be so with heavily interlinked generation busbars, as are often present in systems with a large amount of hydrogeneration concentrated on a few major river systems. With suitable numbering of buses, this Hessian matrix can frequently be made tri-diagonal, and may in some systems be diagonal. Control inequality constraints are handled by Dommel and Tinney's method [11], and penalty factors are used to implement

functional inequality constraints. An interesting feature is the use of a variable factor (≤ 1) which is introduced to damp oscillations and overshooting often caused by the penalty factor method. It is claimed that storage requirements are less than for other similar algorithms.

6.5 THE SEQUENTIAL UNCONSTRAINED MINIMISATIONS TECHNIQUE [7, 8, 10, 25, 26, 27]

This method was applied to thermal dispatch by Sasson [25, 26, 27] and by Dillon and Morsztyn [7, 8, 10]. It consists of transforming the constrained minimisation problem into a sequence of unconstrained minimisations using either interior or exterior penalty functions to attach constraints to the cost function. A new cost function $P(\underline{x}, \underline{r}_k)$ defined by attaching inequality constraints $\underline{g}(\underline{x}) \leq 0$ and equality constraints $\underline{h}(\underline{x}) = 0$ to the original cost function $f(\underline{x})$. Using interior penalty functions

$$P(\underline{x}, \underline{r}_k) = f(\underline{x}) + \sum_i r_{ki} / g_i(\underline{x}) + \sum_j r_{kj}^{-1/2} \cdot h_j^2(\underline{x}) \quad \dots (6.38)$$

or using exterior penalty functions

$$P(\underline{x}, \underline{r}_k) = f(\underline{x}) + \sum_i \frac{1}{r_{ki}} g_i^2(\underline{x}) + \sum_j \frac{1}{r_{kj}} h_j^2(\underline{x}) \quad \dots (6.39)$$

When using exterior penalty functions, only the inequality constraints actually violated are included in P . This augmented cost function is then minimised for a sequence of monotonically decreasing

$r_k > 0$. The constrained minimisation has been replaced by a sequence of unconstrained minimisations which can then be solved using any suitable unconstrained minimisation technique. Sasson [25, 26] initially suggested use of the Fletcher-Powell method, while Dillon and Morsztyn [7, 8, 10] investigated use of both this and a Hessian matrix method. In a later paper, Sasson [27] also suggested use of the Hessian matrix method.

The solution procedure is

Step 1: Solve an initial system load flow.

Step 2: Select initial values for the penalty coefficients r_k .

Step 3: Minimise $P(\underline{x}, r_k)$ using a suitable unconstrained minimisation method.

Step 4: Increase the penalty coefficients for those terms not approaching zero quickly enough and return to Step 3.

It has been found [7] that the Hessian matrix method converges much faster than does the Fletcher-Powell method, and the extreme sparsity of the matrix of second partial derivatives can be used to advantage, improving storage utilisation and speed. The Fletcher-Powell method suffers from round-off error as the system becomes large, leading to convergence difficulties for systems of around 100 nodes.

6.6 THE SEQUENTIAL LINEARLY CONSTRAINED MINIMISATIONS TECHNIQUE

Applied to thermal scheduling by Dillon and Morsztyn [7, 9]

this method employs both projection and penalty functions in defining a transformed problem, and exploits the linear character of many power system constraints. Only nonlinear constraints are included in the augmented cost function which is now defined as

$$P(\underline{x}, \underline{r}_k) = f(\underline{x}) + \sum_i r_{kj}^{\frac{1}{2}} h_{nj}(\underline{x}) \quad \dots \quad (6.40)$$

where h_{nj} are nonlinear constraints only. This function is minimised subject to the linear constraints for a set of positive, monotonically decreasing r_{kj} . Each linearly constrained minimisation is then solved using a projection method. Goldfarb's method [14] which is an extension of the Fletcher-Powell method, and a Hessian matrix method were both applied in ref [9] to minimise each linearly constrained cost function. The method is one of feasible directions, i.e. it starts from a feasible initial point and then locates a new feasible direction which decreases the objective function.

The crux of the method is in determining the projection of the Goldfarb or second-order move vector onto the boundary of the feasible region and then moving in the direction of this projection rather than in the direction of the actual move vectors.

The solution algorithm is as follows

Step 1: Solve an initial load flow.

Step 2: Select a set of penalty constraints r_k for the nonlinear constraints.

Step 3: Solve the linearly constrained problem using Goldfarb's method and check for convergence.

Step 4: Tighten those r_{ki} which do not meet the predetermined tolerance and repeat Step 3.

The method has the important advantage that it provides criteria for removing or adding a limiting constraint to the linear manifold on which projection takes place. This combination of projection and penalty methods reduces the number of constraints relegated to penalty functions and so seeks to avoid their distorting effect on the cost surface. Computationally the method is far superior to the sequential unconstrained minimisation technique, giving large improvements in convergence speed [7].

6.7 METHODS SCHEDULING REACTIVE GENERATION ONLY

Several methods have been developed to optimally schedule the reactive generation of a power system [2, 12, 23, 30], usually minimising transmission losses. Three of these methods are briefly discussed below as they represent part of the scheduling problem in systems with only hydro-generation, and the fourth, that of Peschon et. al. [23] is discussed in 6.4 as it is a precursor of Dommel and Tinney's optimal thermal generation scheduling method [11].

Smith and Tong [30] proposed a one-at-a-time method which involved computing total system losses for three voltage levels, 0.95 pu, 1.00 pu and 1.05 pu at each generation bus in turn while holding voltage levels on the rest of the system constant, fitting a quadratic and moving to its minimum. Being a one-at-a-time method, it is suspected that its convergence was slow and unreliable.

Baumann's method [2] relies on the loss formula expressing the transmission loss P_L in terms of node voltages, active and reactive injections and network driving point and transfer impedances.

Thus

$$P_L = \begin{bmatrix} P^1 & Q^1 \end{bmatrix} \begin{bmatrix} \alpha & -\beta \\ \beta & \alpha \end{bmatrix} \begin{bmatrix} P \\ Q \end{bmatrix} \dots (6.41)$$

Setting $\frac{\partial P_L}{\partial Q_i} = 0$ and using the approximation that all

voltage phase angles are sufficiently close to zero, Baumann obtained an approximate set of necessary conditions for a minimum independent of nodal voltages. The imaginary parts of reactive source currents were then determined from these at each iteration of the load-flow solution, allowing the voltage magnitudes at all nodes except the slack node to vary. While speeding up the load-flow solutions, this method may not converge to the true optimum, and has difficulty implementing constraints on the variables.

Dopazo et. al.'s method [12] of scheduling achieved active dispatch using the conventional co-ordination equations approach described in 2.23. For reactive dispatch, a steepest descent method was used to minimise the transmission losses as expressed by equation (6.41), matrices $[\alpha]$ and $[\beta]$ being updated at each load flow solution. Reactive generations and voltages which reach their bounds during the solution process are held there. This procedure is unsatisfactory as at the optimum, these variables may not be at their limits, and no systematic criterion is given for removal of the variables from their limits.

Dillon [7] suggests that as long as constraints on voltages are not a problem, the reactive optimisation is essentially a problem in quadratic programming. He suggests the use of variable inter-

change or penalty functions to include voltage constraints.

6.8 RECAPITULATION

Six methods for scheduling a thermal power system or the electrical and thermal component of a hydrothermal power system have been presented, together with several methods scheduling reactive generation only.

In the following chapter, Dommel and Tinney's optimal load flow method is discussed fully, and extensions to incorporate area interchange optimisation [32] and automatic adjustment of tap changers and phase shifters [33] within the Newton's method algorithm are made. A number of alternatives for handling functional inequality constraints are discussed, considering both "soft" and "hard" limits.

6.9 REFERENCES

- [1] Abadie, J. and Carpentier, J. "Generalisation of the Wolfe reduced gradient method to the case of nonlinear constraints" in R. R. Fletcher (ed.), Optimisation, Academic press, New York, Chap. 4 p37, 1969.
- [2] Baumann, R. "Power Flow Solution with optimal reactive flow" Archiv fur Electrotech, 48 (1963) 213-244.
- [3] Beale, E. "Applications of mathematical programming techniques", English University Press, 1970.
- [4] Carpentier, J. "Contribution à l'etude du dispatching économique" Bull de la Soc. Fr. des Electriciens v3: (1962) pp 431-447.
- [5] Carpentier, J. and Sirioux, J. "L'Optimisation de la production à Electricite de France" Bull de la Soc. Fr. des Electriciens v4 (1963) pp 121-129.
- [6] Dauphin, S., Feingold, D. and Spohn, G. "Methods of optimising the production of generating stations of a power network" PICA Conf. record 1967 pp 133-140.
- [7] Dillon, T.S. "Problems of optimal economic operation and control of integrated (hydrothermal) and thermal power systems". Ph.D. thesis, Electrical Engineering Department, Monash University, April, 1974.
- [8] Dillon, T.S. and Morsztyn, K. "Application of sequential unconstrained minimisation technique for nonlinear programming to the optimisation of a thermal power system". Proc. 3rd Hawaii International Conf. on System Sciences 1970 pp 949-952.

- [9] Dillon, T.S. and Morsztyn, K. "A new theoretical and computational approach to the exact solution of the problem of optimal control of integrated (hydro-thermal) power systems". 5th IFAC World Congress, Paris, June 1972.
- [10] Dillon, T.S. and Morsztyn, K. "Active and reactive load scheduling in a thermal power system in the presence of tap changing transformers using nonlinear programming" I.E. Aust. Elec. Eng. Trans. v8 No. 2 (1972) pp 67-70.
- [11] Dommel, H.W. and Tinney, W.F. "Optimal power flow solutions", Trans IEEE, Pwr. Appar. Sys. v89 (1968) pp 1866-1874.
- [12] Dopazo, J.F., Klitin, O.A., Stagg, G.W. and Watson, M. "An optimisation technique for real and reactive power allocation" Power Industry Computer Applications Conf. Record, Pittsburgh, 1967, pp 141-154.
- [13] Gamm, A.Z., Krumm, L.A. and Sher, I.A., "Optimising the power flow in a large power system by a gradient method with tearing into subsystems", Elektrichesvo No. 1, 1967 pp 21-29.
- [14] Goldfarb, D., "Extension of Davidon's variable-metric method to maximisation under linear inequality and equality constraints" SIAM J. Appl. Math. v17 (1969) pp 739-764.
- [15] Graves, R.L. and Wolfe, D. "Recent advances in mathematical programming" McGraw-Hill, New York, 1963.
- [16] Griffith, R.E. and Stewart, R.A. "Nonlinear programming technique for optimisation of continuous processing systems" Management Science v7 (1961) pp 379-392.

- [17] Kirchmayer, L.K., Economic Operation of Power Systems, Wiley, New York, 1958.
- [18] Kirchmayer, L.K., Economic Control of Interconnected Systems, Wiley, New York, 1959.
- [19] Kirchmayer, L.K. and Stagg, G.W. "Analysis of total and incremental losses in transmission systems" Trans AIEE v70 (1951) 1 pp 1197-1204.
- [20] Krumm, L.A. "Summary of the gradient method for optimising power flow in an interconnected power system" Isv. Acad. Nauk. USSR No. 3 (1965) pp 3-16.
- [21] Miele, A., Huang, H.V. and Heidemann, J.L. "Sequential Gradient Restoration algorithm for the minimisation of constrained functions - Ordinary and conjugate gradient versions" J. Opt. Theory and appl. v4 (1969) pp 213-243.
- [22] Nabona, N. and Freris, L.L. "Optimisation of economic dispatch through quadratic and linear programming". Proc. IEE v120 No. 5 May 1973.
- [23] Peschon, J., Piercy, D.S., Tinney, W.G., Tsviet, O.J. and Luenod, M. "Optimum control of reactive power flow" Trans IEEE, Pwr. Appar. Sys. v87 (1968) pp 40-48.
- [24] Rashmed, A.M.H. and Kelly, D.H. "Optimal load flow solution using Lagrangian multipliers and the Hessian Matrix". Trans IEEE Pwr. Appar. Sys. v93 (1974) pp 1292-1297.
- [25] Sasson, A.M. "Nonlinear Programming solutions for load flow, minimum loss and economic dispatching problems" Trans IEEE Pwr. Appar. Sys. v88 (1969) pp 399-409.

- [26] Sasson, A.M. "Decomposition techniques applied to the nonlinear programming load flow method". Trans IEEE Pwr. Appar. Sys. v89 (1970) pp 78-82.
- [27] Sasson, A.M., Vittoria, F. and Aboytes, F. "Optimal load flow solutions using the Hessian matrix" IEEE P.I.C.A. Conf. 1971 pp 203-209.
- [28] Sato, N. and Tinney, W.F. "Techniques for exploiting the sparsity of the network admittance matrix". Trans IEEE Pwr. Appar. Sys. v82 (1963) pp 944-950.
- [29] Smirnov, K.A., "Optimisation of the performance of a power system by the decreasing gradient method". Isv. Acad. Nauk. USSR., 1966, No. 2 pp 19-28.
- [30] Smith, H.M. and Tong, S.Y. "Minimising power transmission losses by reactive volt-amp control" Trans IEEE Pwr. Appar. Sys. v82 (1963) pp 542-544.
- [31] Tinney, W.F. and Hart, G.E. "Power flow solution by Newtons method" Trans IEEE Pwr. Appar. Sys. v86 (1967) pp 1449-1460.
- [32] Britton, J.P. "Improved area interchange control for Newton's method load flows" Trans IEEE paper 69 TP124 presented at the 1969 Winter power meeting.
- [33] Meyer, W.S. "New computational techniques for the power flow and economic dispatch problems of electrical utility engineering". University of Minnesota, Ph.D., 1969.

7. OPTIMAL SCHEDULING OF THE ELECTROTHERMAL SUBSYSTEM

By their nature, power system problems are "network problems". For economy and reliability of operation, power systems have evolved so that they generate electrical energy at relatively few points and convey it to its users by means of transmission network or grid; thus the network concept is a physical characteristic of the problem.

The use of Newton's method to solve the nodal-admittance power flow equations (7.1, 7.2) was first treated by Van-Ness [14]. The quadratic convergence was found to be ideal, but direct solution of the linearised equations initially proved far too demanding in memory and run-time to be feasible for large systems. Introduction of optimally ordered triangular factorisation [OOTF] by Sato and Tinney [11], and Carpentier [3] enabled adoption of the Newton power flow, with its superiority over other methods increasing with system size. If all LTC and phase shifter settings are fixed, no area interchanges are specified and no VAR limits placed on generators, one is solving a fixed system; that is one merely solves (7.3) using Newton's method, a process typically requiring four iterations to converge from a flat start (all $V_i = 1 + j.0$) [6, 9]. General power flow solutions are considerably more difficult than such fixed system solutions. The basic solution scheme has been to solve a series of fixed systems, making the required adjustments between such solutions, and repeating until all problem conditions are satisfied [6]. After each such fixed system solution,

- (a) LTC transformers and phase shifter taps are adjusted proportionally to differences between the relevant

scheduled and currently existing problem variables.

(b) Area interchange is adjusted by the displacement of the area slack generators.

(c) Generators which exceed their reactive limits become fixed reactive sources (P-Q nodes) at the appropriate limiting value and are freed (i.e. converted back to P-V nodes) if they back off the limits.

Hopefully, the series of solutions to the fixed-system problems converge so that a valid power flow solution is found. Many complex power systems require 15 or more total Newton iterations. Although Newton's method converges quadratically, such parameter adjustments are at best linear.

Britton [1] showed that the area interchange adjustments could be handled within the Newton algorithm itself by appending additional constraint equations to the load flow equations 7.1 and 7.2. This is satisfactory if the number of areas is small. Meyer [8, 10] extended this procedure to include the automatic adjustment of phase shifters, and also included on-load tap adjustment in the Newton algorithm with no loss of solution speed. Section 7.2 explains these procedures.

With the modifications to Tinney's [11] load flow formulation by Britton [1] and Meyer [8, 10] the load flow calculation for a system in which constraints are not active typically requires four iterations, with 12 a more common number in the presence of active constraints on reactive generation, transformer taps or phase shifter settings.

7.1 THE LOAD-FLOW EQUATIONS

A power system of n nodes (busbars) in steady-state is described by the $2n$ load flow equations:

$$P_{Gi} - P_{Di} - \sum_a \frac{E_i}{Z_{ia}} \{E_a \sin(\phi_i - \phi_a - \delta_{ia}) + E_i \sin \delta_{ia}\} = 0 \quad \dots (7.1)$$

$$Q_{Gi} - Q_{Di} + \sum_a \frac{E_i}{Z_{ia}} \{E_a \cos(\phi_i - \phi_a - \delta_{ia}) - E_i \cos \delta_{ia}\} + Y_{ii} E_i^2 = 0 \quad \dots (7.2)$$

where $\{a\}$ is the set of busbars connected to bus i

P_{Gi}, Q_{Gi} are the active and reactive generation	} at node i
P_{Di}, Q_{Di} are the active and reactive demand	
E_i, ϕ_i are the voltage magnitude and angle	
$Y_{ii} = G_{ii} + jB_{ii}$ is the equivalent admittance to ground	

Z_{ia} is the magnitude of the series impedance	} of line ia
$\delta_{ia} = \sin^{-1} \frac{R_{ia}}{Z_{ia}}$ is the loss angle	

More detailed load flow equations including off-nominal transformers and phase shifters are given by equations (3.3) and (3.4) of Chapter 3.

The set of load flow equations has a unique solution if all but $2n$ of the variables P_{Gi}, Q_{Gi}, E_i and $\phi_i, i = 1, 2, \dots, n$ are fixed. Rewriting (7.1) and (7.2) in control system notation gives a $2n$ - vector equation:

$$\underline{g}(\underline{x}, \underline{u}, \underline{p}) = 0 \quad \dots (7.3)$$

where \underline{u} is the vector of imposed, or control variables

\underline{x} is the vector of dependent, or state variables

\underline{p} is the vector of predetermined or parameter variables, including active and reactive demands P_{Di} and Q_{Di} .

7.2 FEASIBLE SOLUTION TO THE LOAD-FLOW

Expand (7.3) in a Taylor series to first order about an assumed solution $\underline{x}^{(h)}$ and write

$$\begin{aligned}\underline{g}(\underline{x}^{(h+1)}, \underline{u}, \underline{p}) &= \underline{g}(\underline{x}^{(h)} + \underline{\Delta x}, \underline{u}, \underline{p}) \\ &= \underline{g}(\underline{x}^{(h)}, \underline{u}, \underline{p}) + [\underline{g}_x] \cdot \underline{\Delta x} \\ &= \underline{0} \quad \dots (7.4)\end{aligned}$$

where $[\underline{g}_x]$ is the Jacobian matrix of elements $\frac{\partial g_i}{\partial x_j}$ evaluated at assumed solution $\underline{x}^{(h)}$. Superscript (h) indicates the iteration number. The correction

$$\underline{\Delta x} = -[\underline{g}_x]^{-1} \cdot \underline{\Delta g} \quad \dots (7.5)$$

is calculated, and the process is repeated with the new assumed solution

$$\underline{x}^{(h+1)} = \underline{x}^{(h)} + \underline{\Delta x} \quad \dots (7.6)$$

Because of the extreme sparsity of $[\underline{g}_x]$, the inverse can be computed efficiently by optimally ordered triangular factorisation [3, 11, 12, 13] and compressed storage techniques.

7.2.1 Implementation of the Newton Load Flow

To explain the basic concepts and notation of Newton's method power flow computations, consider the sample system of Fig 7.1. Node 6 is assumed to be the swing node, having known voltage

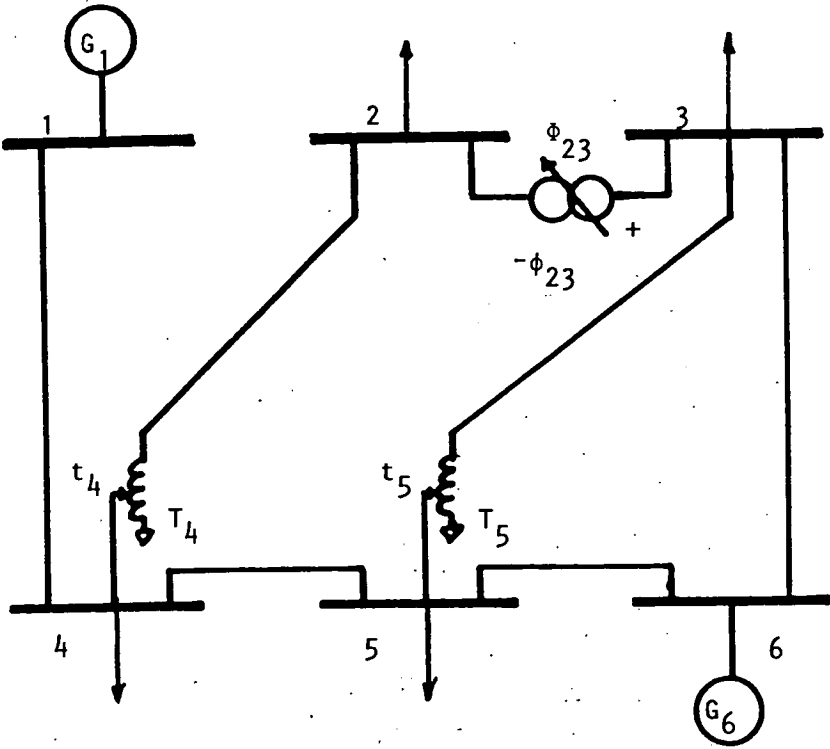


Fig 7.1 Illustrative 6-node Power System [8]

magnitude and angle; node 1 is a generator node with known voltage magnitude and real power injection and all other nodes are load nodes with known real and reactive power injections. T_4 and T_5 are tap changing transformers with tap settings ρ_4 and ρ_5 which are adjusted to regulate voltage magnitudes E_4 and E_5 of buses 4 and 5 at values E_4^{REG} and E_5^{REG} respectively. ϕ_{23} is a phase-shifting transformer with phase shift τ_{23} ; it is adjusted to regulate the real power flow P_{23} in branch 2, 3 to P_{23}^{REG} .

Typical solution methods start by fixing ρ_4 , ρ_5 and τ_{23} at reasonable values, then solving the linearised power flow equations below for node voltage corrections ΔE_i and $\Delta \phi_i$.

$$\begin{array}{c} \Delta \underline{g} \\ \Delta P_1 \\ \Delta P_2 \\ \Delta Q_2 \\ \Delta P_3 \\ \Delta Q_3 \\ \Delta P_4 \\ \Delta Q_4 \\ \Delta P_5 \\ \Delta Q_5 \end{array} = \begin{array}{c} -[g_x] \\ \begin{array}{|c|c|c|c|c|} \hline H_{11} & & & H_{14} & N_{14} \\ \hline & H_{22} & N_{22} & H_{23} & N_{23} \\ & J_{22} & L_{22} & J_{23} & L_{23} \\ \hline & H_{32} & N_{32} & H_{33} & N_{33} \\ & J_{32} & L_{32} & J_{33} & L_{33} \\ \hline H_{41} & H_{42} & N_{42} & & H_{44} & N_{44} \\ J_{41} & J_{42} & L_{42} & & J_{44} & L_{44} \\ \hline & & & H_{53} & N_{53} & H_{54} & N_{54} \\ & & & J_{53} & L_{53} & J_{54} & L_{54} \\ & & & & & H_{55} & N_{55} \\ & & & & & J_{55} & L_{55} \\ \hline \end{array} \end{array} \cdot \begin{array}{c} \Delta \underline{x} \\ \Delta \phi_1 \\ \Delta \phi_2 \\ \Delta E_2/E_2 \\ \Delta \phi_3 \\ \Delta E_3/E_3 \\ \Delta \phi_4 \\ \Delta E_4/E_4 \\ \Delta \phi_5 \\ \Delta E_5/E_5 \end{array}$$

... (7.7)

$$\text{where } H_{ia} = \frac{\partial P_i}{\partial \phi_a}; \quad N_{ia} = \frac{\partial P_i}{\partial E_a} E_a$$

$$J_{ia} = \frac{\partial Q_i}{\partial \phi_a}; \quad L_{ia} = \frac{\partial Q_i}{\partial E_a} E_a$$

... (7.8)

Explicit formulae are:

$$H_{ia} = L_{ia} = - \frac{E_i E_a}{Z_{ia}} \cos (\phi_i - \phi_a - \delta_{ia})$$

$$J_{ia} = - N_{ia} = - \frac{E_i E_a}{Z_{ia}} \sin (\phi_i - \phi_a - \delta_{ia}) \quad \text{for } a \neq i$$

... (7.9)

$$H_{ii} = \sum_{a \neq i} \frac{E_i E_a}{Z_{ia}} \cos (\phi_i - \phi_a - \delta_{ia})$$

$$L_{ii} = - \sum_{a \neq i} \frac{E_i E_a}{Z_{ia}} \cos (\phi_i - \phi_a - \delta_{ia}) + 2 \sum_{a \neq i} \frac{E_i^2}{Z_{ia}} \cos \delta_{ia} - 2 Y_{ii} E_i^2$$

$$N_{ii} = \sum_{a \neq i} \frac{E_i E_a}{Z_{ia}} \sin (\phi_i - \phi_a - \delta_{ia}) + 2 \sum \frac{E_i^2}{Z_{ia}} \sin \delta_{ia}$$

$$J_{ii} = \sum_{a \neq i} \frac{E_i E_a}{Z_{ia}} \sin (\phi_i - \phi_a - \delta_{ia})$$

... (7.10)

This notation is similar to that of Van Ness [14]. After each solution for ΔE_i and $\Delta \phi_i$, tap settings ρ_4 , ρ_5 and τ_{23} would be adjusted by a displacement technique similar to Gausse-Seidel methods to better meet the desired operating conditions:

$$\rho_i^{\text{new}} = \rho_i^{\text{old}} + c_i (E_i^{\text{REG}} - E_i) \quad \text{for } i = 4, 5$$

$$\tau_{23}^{\text{new}} = \tau_{23}^{\text{old}} + d_{23} (P_{23}^{\text{REG}} - P_{23})$$

... (7.11)

Acceleration factors c_4 , c_5 and d_{23} in these relations are empirically determined, often resulting in slow and unpredictable overall convergence.

Meyer [8, 10] suggested that the tap-setting should replace the bus voltage magnitude in the Newton load flow for buses with voltage regulated by tap-changing transformers, i.e. ρ_4 replaces E_4 and ρ_5 replaces E_5 . After incorporating these changes, the relations (7.7) are modified as (7.12) below.

ΔP_1				$H_{14} \ 0$		$\Delta \phi_1$
ΔP_2 ΔQ_2		$H_{22} \ N_{22}$ $J_{22} \ L_{22}$	$H_{23} \ N_{23}$ $J_{23} \ L_{23}$	$H_{24} \ C_{24}$ $J_{24} \ D_{24}$		$\Delta \phi_2$ $\Delta E_2/E_2$
ΔP_3 ΔQ_3		$H_{32} \ N_{32}$ $J_{32} \ L_{32}$	$H_{33} \ N_{33}$ $J_{33} \ L_{33}$		$H_{35} \ C_{35}$ $J_{35} \ D_{35}$	$\Delta \phi_3$ $\Delta E_3/E_3$
ΔP_4 ΔQ_4	H_{41} J_{41}	$H_{42} \ N_{42}$ $J_{42} \ L_{42}$		$H_{44} \ C_{44}$ $J_{44} \ D_{44}$	$H_{45} \ 0$ $J_{45} \ 0$	$\Delta \phi_4$ $\Delta \rho_4/\rho_4$
ΔP_5 ΔQ_5			$H_{53} \ N_{53}$ $J_{53} \ L_{53}$	$H_{54} \ 0$ $J_{54} \ 0$	$H_{55} \ C_{55}$ $J_{55} \ D_{55}$	$\Delta \phi_5$ $\Delta \rho_5/\rho_5$

$$\text{where: } C_{ia} = \frac{\partial P_i}{\partial \rho_a} \rho_a ; \quad D_{ia} = \frac{\partial Q_i}{\partial \rho_a} \rho_a \quad \dots (7.12)$$

$$\dots (7.13)$$

$$\text{or } C_{ia} = \frac{E_i E_a}{Z_{ia}} \sin (\phi_i - \phi_a - \delta_{ia})$$

$$D_{ia} = - \frac{E_i E_a}{Z_{ia}} \cos (\phi_i - \phi_a - \delta_{ia})$$

for $a \neq i$

$$\dots (7.14)$$

$$\begin{aligned}
 C_{aa} &= 2\rho_a^2 G_{aa} E_a^2 + \frac{E_i E_a}{Z_{ia}} \sin(\phi_i - \phi_a - \delta_{ia}) \\
 D_{aa} &= -2\rho_a^2 B_{aa} E_a^2 - \frac{E_i E_a}{Z_{ia}} \cos(\phi_a - \phi_i - \delta_{ia}) \\
 &\dots (7.15)
 \end{aligned}$$

where it is assumed that node a is regulated off node i as in Fig 7.1.

Terms C_{ia} and D_{ia} replace the usual N_{ia} and L_{ia} respectively.

Because the variable ρ_a is not a node variable in the same sense as E_a , some new zeros in equation (7.12) are seen; C_{ia} and D_{ia} are not only always zero where no branch ia exists, but also where branch ia does not represent the tap-changing transformer regulating node a.

Each tap-changing transformer-regulated node column in equation (7.12) has only one non-zero off-diagonal C_{ia} and D_{ia} term. This enables solution of systems including tap-changer adjustments in typically four iterations if VAR limit problems are not encountered. Peterson [10] verified this method for a 347 node system.

Meyer [8, 10] also introduced a system for adjustment of phase shifter taps based on Britton's [1] basic concept for interarea control. For the system of Fig 7.1, the phase shifter ϕ_{23} is typically used to constrain the power flow, P_{23} , in branch 23 to same value P_{23}^{REG} . Since P_{23} is not a regular problem variable, a variable exchange such as that used to handle tap-changing transformers is not possible. The phase setting τ_{23} enters as an extra variable, and the linearised version of

$$0 = -P_{23}^{REG} + P_{23} \quad \dots (7.16)$$

can be introduced as a further constraint on the power flow equations. The resulting linearised equations appear in (7.17) where the tap-changing transformer adjustments remain.

ΔP_1	H_{11}			$H_{14} \ 0$			$\Delta \phi_1$
ΔP_2	$H_{22} \ N_{22}$	$H_{23} \ N_{23}$	$H_{24} \ C_{24}$		$R_{2, 23}$		$\Delta \phi_2$
ΔQ_2	$J_{22} \ L_{22}$	$J_{23} \ L_{23}$	$J_{24} \ D_{24}$		$S_{2, 23}$		$\Delta E_2/E_2$
ΔP_3	$H_{32} \ N_{32}$	$H_{33} \ N_{33}$		$H_{35} \ C_{35}$	$R_{3, 23}$		$\Delta \phi_3$
ΔQ_3	$J_{32} \ L_{32}$	$J_{33} \ L_{33}$		$J_{35} \ D_{35}$	$S_{3, 23}$		$\Delta E_3/E_3$
ΔP_4	H_{41}	$H_{42} \ N_{42}$		$H_{44} \ C_{44}$	$H_{45} \ 0$		$\Delta \phi_4$
ΔQ_4	J_{41}	$J_{42} \ L_{42}$		$J_{44} \ D_{44}$	$J_{45} \ 0$		$\Delta \rho_4/\rho_4$
ΔP_5			$H_{53} \ N_{53}$	$H_{54} \ 0$	$H_{55} \ C_{55}$		$\Delta \phi_5$
ΔQ_5			$J_{53} \ L_{53}$	$J_{54} \ 0$	$J_{55} \ D_{55}$		$\Delta \rho_5/\rho_5$
ΔP_{23}		$F_{23, 2} \ G_{23, 2}$	$F_{23, 3} \ G_{23, 3}$			$A_{23, 23}$	$\Delta \tau_{23}$

... (7.17)

where $R_{i, ia} = \partial P_i / \partial \tau_{ia}$; $S_{i, ia} = \partial Q_i / \partial \tau_{ia}$

$A_{ia, ia} = \partial P_{ia} / \partial \tau_{ia}$; $F_{ia, j} = \partial P_{ia} / \partial \phi_j$

$R_{a, ia} = \partial P_a / \partial \tau_{ia}$; $S_{a, ia} = \partial Q_a / \partial \tau_{ia}$

$G_{ia, j} = \frac{\partial P_{ia}}{\partial E_j} \cdot E_j$

... (7.18)

or $G_{ia, a} = S_{i, ia} = -S_{a, ia} = \frac{E_i E_a}{Z_{ia}} \sin (\phi_i - \phi_a - \delta_{ia})$

$R_{i, ia} = R_{a, ia} = \frac{E_i E_a}{Z_{ia}} \cdot \cos (\phi_i - \phi_a - \delta_{ia})$

$F_{ia, i} = -F_{ia, a} = A_{ia, ia} = R_{i, ia}$

$G_{ia, i} = 2 E_i^2 G_{ii} + G_{ia, a}$

... (7.19)

These formulae apply for a phase shifter ϕ_{ia} in branch ia as in Fig 7.1. Reference [8] reported numerical tests on basic five-node and 20-node systems and concluded that systems can, by this method, have phase shifter adjustments made automatically within the Newton load flow algorithm which still converges at about the same rate as the corresponding fixed system, that is, in about four iterations.

Because extra rows and columns are being added to the Jacobian representing the fixed system, triangularisation and back-substitution must necessarily take longer and require additional memory. Most systems have very few, if any, phase shifters so the effect on sparsity of numerous phase shifters is not relevant.

Tinney and Hart [12] suggested that voltage controller busbars have their reactive generation checked for limit violation, and when the limit is reached, converting the node to reactive generation control.

Tinney and Hart [12] suggested that limits may be imposed on voltage at a controlled reactive generation node, and on reactive generation at a voltage controlled node by allowing corrections in the control variable until the limit is reached, then converting the node type. On reaching its voltage limit, a controlled reactive generation node would have its voltage fixed at the limit value, with reactive generation then becoming variable. Later it may be allowed to vary again should this be justified.

Large scale testing by Peterson and Meyer [10] indicates that transformer tap limits are well behaved, and limits may be imposed by setting all taps ρ_j in the feasible range and allowing corrections $\Delta\rho_j$ until the limit is reached, then converting the

tap-changer into a fixed transformer. This may be allowed to vary again should this be justified by a change in voltage E_j which is being monitored.

7.3 OPTIMAL LOAD FLOW WITHOUT INEQUALITY CONSTRAINTS

The objective function to be minimised subject to equality constraints (7.3) can now be defined as

$$f = \sum_{i_1} K_i \{P_{Gi}\}, \quad k_i \text{ constant} \quad \dots (7.20)$$

where $\{i_1\}$ is the set of buses with thermal generation, or if reactive generation only is to be optimised, as in an all-hydro system,

$$\begin{aligned} f &= \text{total system transmission losses} \\ &= \sum_i \sum_a \frac{E_i}{Z_{ia}} [E_a \cdot \sin(\phi_i - \phi_a - \delta_{ia}) + E_i \cdot \sin \delta_{ia}] \end{aligned} \quad \dots (7.21)$$

Utilising Lagrange multipliers, $\inf_{\underline{u}} f(\underline{x}, \underline{u})$ subject to equality constraints (7.3) is found by introducing as many auxiliary variables λ_i as there are equality constraints, and minimising the resulting Lagrangian

$$L(\underline{x}, \underline{u}, \underline{p}) = f(\underline{x}, \underline{u}) + \underline{g}^T(\underline{x}, \underline{u}, \underline{p}) \cdot \underline{\lambda} \quad \dots (7.22)$$

From (7.9) we obtain the necessary conditions for an unconditional minimum:

$$\underline{L}_{\underline{x}} = \underline{f}_{\underline{x}} + [\underline{g}_{\underline{x}}]^T \cdot \underline{\lambda} = \underline{0} \quad \dots (7.23)$$

$$\underline{L}_{\underline{u}} = \underline{f}_{\underline{u}} + [\underline{g}_{\underline{u}}]^T \cdot \underline{\lambda} = \underline{0} \quad \dots (7.24)$$

$$\underline{L}_{\underline{\lambda}} = \underline{g}(\underline{x}, \underline{u}, \underline{p}) = \underline{0} \quad \dots (7.25)$$

For any feasible but not yet optimal load flow, (7.3) is satisfied and $\underline{\lambda}$ is obtained from (7.10) since $[g_x]$, the Jacobian of the load-flow solution, has been computed. The remaining condition, $\underline{L}_u = \underline{0}$, is not satisfied. \underline{L}_u is the gradient vector of the Lagrangian with respect to \underline{u} , orthogonal to the local contour of the objective function. Equations (7.23), (7.24) and (7.3) are nonlinear in \underline{x} and \underline{u} , and may be solved iteratively.

7.4 INEQUALITY CONSTRAINTS ON CONTROL VARIABLES

In practice, permissible values of the control variables are constrained

$$\underline{\hat{u}} \leq \underline{u} \leq \underline{\hat{u}} \quad \dots \quad (7.26)$$

These inequality constraints may be enforced by ensuring that the adjustment algorithm (7.25) does not send any control variable beyond its permissible limit, that is,

$$u_i(h+1) = \begin{cases} \hat{u}_i & \text{if } u_i(h) + \Delta u_i > \hat{u}_i \\ \underline{\hat{u}}_i & \text{if } u_i(h) + \Delta u_i < \underline{\hat{u}}_i \\ u_i(h) + \Delta u_i & \text{otherwise} \end{cases} \quad \dots \quad (7.27)$$

When a control variable has reached its limit, the corresponding gradient component must still be computed in the following cycles, as it may eventually back off the constraint. The step then proceeds along the projection of the negative gradient onto the active constraints. This is the gradient-projection method. At the minimum, the components of $\underline{\nabla f}$ will be

$$\begin{aligned}
 \frac{\delta f}{\delta u_i} &= 0 \quad \text{if } \check{u}_i < u_i < \hat{u}_i \\
 &\leq 0 \quad \text{if } u_i = \hat{u}_i \\
 &\geq 0 \quad \text{if } u_i = \check{u}_i
 \end{aligned}
 \quad \dots \quad (7.28)$$

The Kuhn-Tucker theorem shows that (7.17) are sufficient conditions for a minimum provided the functions involved are convex.

By making the voltage at the slack node a bounded control variable, this node no longer determines the general system voltage level. Its only role is to take up the generation to balance system losses. This approach may ease overvoltage problems at some system nodes, or allow greater system economy.

7.5 FUNCTIONAL INEQUALITY CONSTRAINTS

Besides inequality constraints on controls \underline{u} , there can also be functional inequality constraints

$$\underline{h}(\underline{x}, \underline{u}) \leq 0 \quad \dots \quad (7.29)$$

of which the most common form is

$$\underline{\check{x}} \leq \underline{x} \leq \underline{\hat{x}} \quad \dots \quad (7.30)$$

These are difficult to handle. A new search direction different from the negative gradient must be formed when confronting a functional constraint.

Possible methods are:

- (a) Linearise the problem when a boundary is encountered (approximate method) and use linear programming techniques.

- (b) Transform the problem formulation so that "hard" functional constraints become control variable constraints, and some "soft" control variable constraints become functional constraints [2, 4, 6]. For example, replace $x_i - u_i < 0$, a "hard" constraint and $\check{u}_j \leq u_j \leq \hat{u}_j$, a "soft" constraint, by $u_j' \triangleq x_i - u_j \leq 0$ and $\check{u}_j \leq x_i - u_j' \leq \hat{u}_j$
- (c) Penalty methods.
- (d) Sequential unconstrained minimisation (SUMT) [5, 7].
- (e) Sequential linearly constrained minimisation [5, 7].

Methods (a), (b) and (c) require the sensitivity matrix at each step and hence involve a large amount of computing. The constraints enforced by (c), (d) and (e) are "soft", but (b) handles hard constraints explicitly. It has been found [6] that there is little practical disadvantage incurred by using "soft" constraints where physical "hard" constraints in fact exist.

7.5.1 External Penalty Function Method

External penalty functions may be used to advantage in the search, because

- (a) functional constraints in a real system are seldom rigid limits in the mathematical sense, but are soft limits as imposed by penalty techniques.
- (b) such methods add little to the algorithm, only adding terms to f_x and f_u . This may increase the number of iterations for convergence due to distortion of the cost surface.

(c) they produce a feasible power flow solution with penalties indicating trouble spots such as under-or-over-voltage nodes where inappropriate rigid limits could exclude solution.

We replace the objective function by

$$f_{\text{pen}} = f(\underline{x}, \underline{u}) + \sum W_j \quad \dots (7.31)$$

where a penalty W_j is introduced for each violated constraint. Often

$$W_j = \begin{cases} r_j (x_j - \hat{x}_j)^2 & \text{if } x_j > \hat{x}_j \\ r_j (x_j - \check{x}_j)^2 & \text{if } x_j < \check{x}_j \\ 0 & \text{otherwise} \end{cases} \quad \dots (7.32)$$

The steeper the penalty function (greater r_j) the closer the solution to a rigid limit, but in the extreme, instability may result. An effective solution is to start with low r_j and increase it during the optimisation process if the solution exceeds a certain limit. The effect of penalty functions is to close the contours, with the minimum then lying in unconstrained space.

The penalty method is not invariant to choice of control variables. An important example of this is at nodes with reactive power control, for which we must observe

$$\check{E}_i \leq E_i \leq \hat{E}_i \quad \dots (7.33)$$

$$\check{Q}_{Gi} \leq Q_{Gi} \leq \hat{Q}_{Gi} \quad \dots (7.34)$$

one a control constraint and the other a functional constraint. In this case E_i is best chosen as control variable because

(a) a Newton power flow (polar form) involves only one equation for a PV node, but two for a PQ node.

(b) the limits on voltage are more severe than those on VAR generation, since the latter can be overloaded for short periods.

Another approach is to transform a PV node into a PQ node when the boundary is encountered. This is utilised in computations in this thesis.

7.5.2 The Sequential Unconstrained Minimisation Technique [7]

This is an interior penalty function method formulated to handle both equality and inequality constraints.

SUMT transforms the constrained problem

$$\begin{aligned} \min \{ & f(\underline{x}, \underline{u}, \underline{p}) / h_i(\underline{x}, \underline{u}, \underline{p}) > 0, i = 1, \dots, q; \\ & g_j(\underline{x}, \underline{u}, \underline{p}) = 0, j = 1, \dots, \ell \} \end{aligned} \quad \dots (7.35)$$

into a series of unconstrained minimisation problems using penalty functions to impose constraints. To achieve this, we define a modified objective function

$$\begin{aligned} \mathcal{F}(\underline{x}, \underline{u}, \underline{p}, r) = & f(\underline{x}, \underline{u}, \underline{p}) - r \sum_{i=1}^q \log_e [h_i(\underline{x}, \underline{u}, \underline{p})] \\ & + \frac{1}{r} \sum_{j=1}^{\ell} g_j^2(\underline{x}, \underline{u}, \underline{p}) \end{aligned} \quad \dots (7.36)$$

where r is a weighting coefficient. The algorithm is

- (a) an initial state \underline{x}^0 within the admissible region is chosen so that $h_i(\underline{x}^0, \underline{u}^0, \underline{p}^0) > 0; i = 1, \dots, q;$
- ... (7.37)

- (b) an initial value of r , say $r^0 = 1$, is chosen.
- (c) the function $\mathcal{F}(\underline{x}, \underline{u}, \underline{p}, r)$ is minimised starting from \underline{x}^0 by any suitable unconstrained minimisation method. The minimum of $\mathcal{F}(\underline{x}, \underline{u}, \underline{p}, r^0)$ is assumed to occur at \underline{x}^1 .
- (d) a new value of r is computed; in general

$$r^{(h+1)} = r^{(h)}/k; \quad k > 1 \quad \dots \quad (7.38)$$

with k normally around 4 or 5, and for stability $k < 10$.

The computation is terminated when the change in \mathcal{F} is below a certain limit.

A rigorous proof of convergence, when $f(\underline{x}, \underline{u}, \underline{p})$ and all the $g_j(\underline{x}, \underline{u}, \underline{p})$'s are concave, has been given by Fiacco and McCormick [7].

The method may be applied selectively, the equality constraints and some of the functional constraints being handled in more appropriate ways. Again this method adds little to the algorithm, but may slow the convergence.

7.6 RECAPITULATION

The optimal power flow technique adopted for use in tests is based on the Newton's method power flow, with modifications to Dommel and Tinney's [6] formulation suggested by Meyer [8, 10] and described in 7.2.1. This was programmed in FORTRAN on the Burroughs B6700 of the University of Tasmania. Optimally ordered triangular factorisation following Tinney and Hart's scheme 2 [12] for optimal ordering was used. In this, the nodes are renumbered so that at each step of the elimination, the next node to be eliminated is the one having fewest connected branches. This method requires simulation of the elimination process to take into account changes in the node-

branch connections. Tinney and Hart's scheme 3 [12] in which the nodes are renumbered so that at each step of the elimination the next node to be eliminated is the one which will introduce the fewest new equivalent branches, may have some advantage over scheme 2 for use in hydrothermal optimisation even though it is much more involved, requiring simulation of every feasible alternative at each step. This is because several optimal load flow solutions are required in each time interval and most time intervals will pertain to identical transmission systems. Scheme 3 was not tested because of added complexity of programming.

For a test system of 25 buses, 35 lines, 5 generators and 4 transformers, of which two were used to regulate their associated bus voltages, the optimal ordering process (scheme 2) was found to take 3.3 seconds and the power flow was found to take 0.38 seconds per iteration. With careful attention to programming, a substantially reduced execution time would be expected.

The optimisation of system thermal generation, VAR generation and transformer and phase shifter taps may proceed by any method directly involving only first order sensitivity information, such as the hill-climbing techniques described in Chapter 5 for use in scheduling the hydro subsystem. The method adopted was optimal gradient which, without active inequality constraints takes about 18 Newton iterations to converge from a flat start. The sensitivity information required by the scheduling method for the hydro subsystem and coordination is computed in the course of the optimisation with little extra work as described in Chapters 8 and 9.

Because first order gradient methods are not invariant to scaling, and also for convenience, per-unit quantities are used throughout the load flow.

Any method of optimal load flow generating the required sensitivity information with sufficient speed and accuracy could be utilised in the overall hydrothermal optimal control computation.

7.7 REFERENCES

- [1] Britton, J.P. "Improved area interchange control for Newton's method load flows". Trans. IEEE, paper 69TP124 presented at the 1969 Winter Power Meeting.
- [2] Carpentier, J. "Contribution à l'étude du dispatching économique". Bul. Soc. Franc. Elect., Ser. 8, vol. 3, pp 431-447, August 1962.
- [3] Carpentier, J. "Ordered Eliminations". Proc. power system computation conference, London 1963.
- [4] Carpentier, J. and Sirieux, J. "L'Optimisation de la production à l'Electricite de France", Bul. Soc. Franc. Elect., IV pp 121-129 March 1963.
- [5] Dillon, T.S. "Problems of optimal economic operation and control of integrated (hydro-thermal) and thermal power systems". Monash University, Ph. D. 1974.
- [6] Dommel, H.W. and Tinney, W.F. "Optimal Power flow solutions". Trans. IEEE pwr. app. and sys. V. 87 No. 10 pp 1866-1876, Oct. 1968.
- [7] Fiacco, A.V. and McCormick, G.P. "Nonlinear programming: Sequential unconstrained minimisation techniques", John

Wylie and Sons, New York, 1968.

- [8] Meyer, W.S. "New Computational techniques for the power flow and economic dispatch problems of electric utility engineering". University of Minnesota, Ph. D, 1969.
- [9] Peschon, J., Piercy, D.S., Tinney, W.F., Tsviet, O.J. and Cuenod, M. "Optimal control of reactive power flow". Trans. IEEE on Pwr. app. and sys., V. 87, pp 40-48, Jan. 1968.
- [10] Peterson, N.M. and Meyer, W.S. "Automatic adjustment of transformer and phase shifter taps in the Newton power flow". Trans. IEEE Power meeting, Winter 1970.
- [11] Sato, N. and Tinney, W.F. "Techniques for exploring the sparsity of the network admittance matrix". Trans. IEEE on pwr. app. and sys. V. 82 pp 944-950.
- [12] Tinney, W.F., and Hart, C.E. "Power flow solution by Newton's method". Trans. IEEE on power apparatus and systems, V. 86 No. 11 pp 1449-1460, November, 1967.
- [13] Tinney, W.F. and Walker, J.W. "Direct solutions of sparse network equations by optimally ordered triangular factorisation". Proc. IEEE. V. 55, No. 11 pp 1801-1809, Nov. 1967.

- [14] Van-Ness, J.E. and Griffin, J.H. "Elimination methods for load flow studies". Trans. AIEE on pwr. app. and sys. V.80 pp 299-304, 1961.

8. THE INTEGRATED SYSTEM

As stated previously, scheduling of a power system which includes hydro plants is a variational problem of infinite dimension in the time-domain.

This integrated problem may be solved via a sequence of lower dimensional problems, alternately adjusting the schedules of hydro plants using a local approximation to optimal operation of the electrothermal subsystem and only thermal plants with fixed hydrogeneration, both with reference to the same goal.

Chapter 5 outlined such a decomposition method in which an initial, feasible suboptimal schedule, generated by the step-loading technique developed in chapter 4, is refined, employing optimal load flow computations to schedule thermal generation, VAR allocation and regulator settings, alternating with a suitable hill-climber using sensitivity information derived from the set of optimal load flow results to improve the hydro subsystem schedule. Six hill-climbers were tested by Monte-Carlo simulations for their performance over a small number of steps. Two methods, one gradient type and one Newton type were found to be superior, the former in terms of ability to converge under almost all conditions, and the latter for its extremely fast convergence on near-quadratic hills.

The decomposition method for refinement of the step-loading schedule for a power system which contains hydro plants is developed in detail in this chapter.

8.1 REPRESENTATION OF THE ELECTROTHERMAL SUBSYSTEM

Tests, documented in chapter 5, on a selection of hill-climber algorithms for coordination of the hydro subsystem and electrothermal subsystems were based on random deviations from a perfectly quadratic hill. To determine whether a gradient or Newton algorithm is more appropriate, the shape of the performance index in relation to changes in hydro plant output must be established.

Looking ahead to section 9.1, which describes sensitivity relations in the electrothermal subsystem, after an optimal load flow solution has been found and hydro plant generations, considered as parameters in the optimal load flow computations, are altered, an accurate approximation to the new optimal schedule may be found without obtaining a new optimal load flow solution. This schedule takes account of both equality and inequality constraints acting on both control and state electrical variables. Specifically, equation (9.40) states

$$\begin{bmatrix} [L_{xx}] & [L_{xu}] & [g_x]^T & [1] & [-1] & [0] & [0] \\ [L_{ux}] & [L_{uu}] & [g_u]^T & [0] & [0] & [1] & [-1] \\ [g_x] & [g_u] & [0] & [0] & [0] & [0] & [0] \\ [\hat{u}] & [0] & [0] & [x-\hat{x}] & [0] & [0] & [0] \\ [-\hat{u}] & [0] & [0] & [0] & [\tilde{x}-x] & [0] & [0] \\ [0] & [\hat{v}] & [0] & [0] & [0] & [u-\hat{u}] & [0] \\ [0] & [-\hat{v}] & [0] & [0] & [0] & [0] & [\tilde{u}-u] \end{bmatrix} \begin{bmatrix} \underline{\delta x} \\ \underline{\delta u} \\ \underline{\delta \lambda} \\ \underline{\delta \mu} \\ \underline{\delta \tilde{u}} \\ \underline{\delta \tilde{v}} \\ \underline{\delta \tilde{v}} \end{bmatrix} = - \begin{bmatrix} [L_{xp}] \\ [L_{up}] \\ [g_p] \\ [(x-\hat{x})_p] \\ [(\tilde{x}-x)_p] \\ [(u-\hat{u})_p] \\ [(\tilde{u}-u)_p] \end{bmatrix} \cdot \underline{\delta p}$$

where $[\hat{u}] = \begin{bmatrix} u_1 & 0 & 0 & 0 \\ 0 & u_2 & 0 & 0 \\ 0 & 0 & & \\ 0 & 0 & & u_n \end{bmatrix}$ etc.

... (8.1)

Writing this as

$$[A] \cdot \begin{bmatrix} \underline{\delta x} \\ \underline{\delta u} \\ \underline{\delta \lambda} \\ \underline{\hat{\delta \mu}} \\ \underline{\check{\delta \mu}} \\ \underline{\hat{\delta v}} \\ \underline{\check{\delta v}} \end{bmatrix} = - [B] \cdot \underline{\delta p} \quad \dots (8.2)$$

we can express the vector of primal and dual variables as

$$\begin{bmatrix} \underline{\delta x} \\ \underline{\delta u} \\ \underline{\delta \lambda} \\ \underline{\hat{\delta \mu}} \\ \underline{\check{\delta \mu}} \\ \underline{\hat{\delta v}} \\ \underline{\check{\delta v}} \end{bmatrix} = - [A]^{-1} \cdot [B] \cdot \underline{\delta p} \quad \dots (8.3)$$

Separating this into components

$$\begin{aligned} \underline{\delta x} &= [T_1] \cdot \underline{\delta p} \\ \underline{\delta u} &= [T_2] \cdot \underline{\delta p} \\ \underline{\delta \lambda} &= [T_3] \cdot \underline{\delta p} \\ \underline{\hat{\delta \mu}} &= [T_4] \cdot \underline{\delta p} \\ \underline{\check{\delta \mu}} &= [T_5] \cdot \underline{\delta p} \\ \underline{\hat{\delta v}} &= [T_6] \cdot \underline{\delta p} \\ \underline{\check{\delta v}} &= [T_7] \cdot \underline{\delta p} \end{aligned} \quad \dots (8.4)$$

The performance index $F(\underline{x}, \underline{u})$ is a function of the control and state variables, but not directly of the hydro plant outputs, which are defined as parameters in the optimal load flow computations. Expanding a deviation δF in the performance index F as a Taylor series to second order,

$$\begin{aligned} \delta F = & \underline{F}_x^T \cdot \underline{\delta x} + \underline{F}_u^T \cdot \underline{\delta u} \\ & + \frac{1}{2} \underline{\delta x}^T \cdot [F_{xx}] \cdot \underline{\delta x} + \frac{1}{2} \underline{\delta u}^T \cdot [F_{uu}] \cdot \underline{\delta u} \\ & + \underline{\delta x}^T \cdot [F_{xu}] \cdot \underline{\delta u} \end{aligned} \quad \dots (8.5)$$

Substituting from (8.4) into (8.5)

$$\begin{aligned} \delta F = & \underline{F}_x^T \cdot [T_1] \cdot \underline{\delta p} + \underline{F}_u^T \cdot [T_2] \cdot \underline{\delta p} \\ & + \frac{1}{2} \underline{\delta p}^T [T_1]^T [F_{xx}] \cdot [T_1] \cdot \underline{\delta p} \\ & + \frac{1}{2} \underline{\delta p}^T [T_2]^T [F_{uu}] \cdot [T_2] \cdot \underline{\delta p} \\ & + \underline{\delta p}^T [T_1] \cdot [F_{xu}] \cdot [T_2] \cdot \underline{\delta p} \\ = & (\underline{F}_x^T \cdot [T_1] + \underline{F}_u^T \cdot [T_2]) \cdot \underline{\delta p} \\ & + \underline{\delta p}^T \cdot (\frac{1}{2} [T_1]^T [F_{xx}] \cdot [T_1] + \frac{1}{2} [T_2]^T [F_{uu}] \cdot [T_2] \\ & + [T_1]^T [F_{xu}] \cdot [T_2]) \cdot \underline{\delta p} \end{aligned} \quad \dots (8.6)$$

Which is of the form

$$\delta F = \underline{r}^T \cdot \underline{\delta p} + \underline{\delta p}^T \cdot [R] \cdot \underline{\delta p} \quad \dots (8.7)$$

where \underline{r} and $[R]$ are evaluated at the optimum, defined by $(\underline{x}^*, \underline{u}^*, \underline{p}^*)$.

Equation (8.6) ignores the important second order terms

$$\begin{aligned} & \underline{\delta p}^T \cdot [\underline{F}_x^T \cdot [\underline{x}_{pp}]] \cdot \underline{\delta p} \\ \text{and } & \underline{\delta p}^T \cdot [\underline{F}_u^T \cdot [\underline{u}_{pp}]] \cdot \underline{\delta p} \\ & \dots \quad (8.8) \end{aligned}$$

where $[\underline{x}_{pp}]$ and $[\underline{u}_{pp}]$ represent the three dimensional matrix of second derivatives of the control and state vectors with respect to the parameter vector.

If these additional terms are included in (8.6) the form of equation (8.7) is unchanged although the terms of matrix $[R]$ will change in value. Confining variations in the parameters \underline{p} to variations in the real power output \underline{P}_H of the hydro plants excluding the slack bus plant, the instantaneous objective function may be considered as a quadratic function of \underline{P}_H if the electrothermal subsystem is operated optimally.

Since we are now optimising the hydro subsystem, we wish to redefine the set of control, state and parameter vectors \underline{x} , \underline{u} and \underline{p} for this new problem.

As outlined in chapter 5, the decomposition technique relies on application of the Kuhn-Tucker necessary conditions. For this, the variational problem must be replaced by a series of static problems corresponding to a discrete set of time intervals, together with overall flow constraints. These problems are then combined to form a single constrained static optimisation.

Replacing the parameters vector \underline{p} in equation (8.7) by \underline{P}_{Hk} , the vector of hydro plant outputs in time interval k , we have, for time interval k

$$\delta F_k = \underline{r}_k^T \cdot \underline{\delta P}_{Hk} + \underline{\delta P}_{Hk}^T \cdot [\underline{R}_k] \cdot \underline{\delta P}_{Hk} \quad \dots (8.9)$$

where F_k is the performance index, or cost rate, in time interval k and \underline{r}_k , $[\underline{R}_k]$ are defined by (8.7) for time interval k .

In the hydro subsystem, the control vector is the vector of discharge rates \underline{u}_k for hydro plants in each time interval $k = 1, 2, \dots, m$.

Since unit commitment is fixed by the step-loading schedule, the output of each hydro plant is a smooth near-linear function of the plant discharge rate. The nonlinearity present is detailed in section 3.4.1 and is due to water conduit and turbine losses, both near-quadratic functions of discharge rate. The output P_{Hik} of a hydro plant i in time interval k may therefore be accurately described by a Taylor series expansion to second order terms. Therefore

$$\delta P_{Hik} = \partial P_{Hik} / \partial u_{ik} \cdot \delta u_{ik} + \frac{1}{2} \partial^2 P_{Hik} / \partial u_{ik}^2 \cdot \delta u_{ik}^2 \quad \dots (8.10)$$

with the first order term dominant. u_{ik} is the discharge rate of hydro plant i in time interval k . In vector notation

$$\underline{\delta P}_{Hk} = \left[\underline{P}_{Hk_u} \right] \cdot \underline{\delta u}_k + \frac{1}{2} \underline{\delta u}_k^T \left[\underline{P}_{Hk_{uu}} \right] \cdot \underline{\delta u}_k \quad \dots (8.11)$$

Substituting this into (8.9) and rearranging,

$$\begin{aligned}
 \delta F_k &= \underline{r}_k^T \cdot [\underline{P}_{Hk_u}] \cdot \underline{\delta u}_k \\
 &+ \underline{\delta u}_k^T \cdot \left\{ [\underline{P}_{Hk_u}]^T \cdot [\underline{R}_k] \cdot [\underline{P}_{Hk_u}] + \frac{1}{2} \underline{r}_k \cdot [\underline{P}_{Hk_{uu}}] \right\} \cdot \underline{\delta u}_k \\
 &+ \frac{1}{2} \underline{\delta u}_k^T \cdot [\underline{P}_{Hk_{uu}}]^T \cdot \underline{\delta u}_k \cdot [\underline{R}_k] \cdot [\underline{P}_{Hk_u}] \cdot \underline{\delta u}_k \\
 &+ \frac{1}{2} \underline{\delta u}_k^T \cdot [\underline{P}_{Hk_u}]^T \cdot [\underline{R}_k] \cdot \underline{\delta u}_k \cdot [\underline{P}_{Hk_{uu}}] \cdot \underline{\delta u}_k \\
 &+ \frac{1}{4} \underline{\delta u}_k \cdot [\underline{P}_{Hk_{uu}}]^T \cdot \underline{\delta u}_k \cdot [\underline{R}_k] \cdot \underline{\delta u}_k \cdot [\underline{P}_{Hk_{uu}}] \cdot \underline{\delta u}_k \\
 &\dots \quad (8.12)
 \end{aligned}$$

Since we are using an iterative process, recalculating $[\underline{R}_k]$ and \underline{r}_k at each iteration, and since \underline{P}_{Hk} is a nearly linear function of \underline{u}_k , we can ignore the third and fourth order terms of (8.12) which then becomes

$$\begin{aligned}
 \delta F_k &\approx \underline{r}_k^T \cdot [\underline{P}_{Hk_u}] \cdot \underline{\delta u}_k \\
 &+ \underline{\delta u}_k^T \cdot \left\{ [\underline{P}_{Hk_u}]^T \cdot [\underline{R}_k] \cdot [\underline{P}_{Hk_u}] + \frac{1}{2} \underline{r}_k \cdot [\underline{P}_{Hk_{uu}}] \right\} \cdot \underline{\delta u}_k \\
 &\dots \quad (8.13)
 \end{aligned}$$

In terms of first and second derivatives of the performance index, this can be rewritten

$$\begin{aligned}
 \delta F_k &= \underline{F}_{k_u}^T \cdot \underline{\delta u}_k + \frac{1}{2} \underline{\delta u}_k^T \cdot [\underline{F}_{k_{uu}}] \cdot \underline{\delta u}_k \\
 &\dots \quad (8.14)
 \end{aligned}$$

$$\begin{aligned}
 \text{where } \underline{F}_{k_u}^T &= \underline{r}_k^T \cdot \underline{P}_{Hk_u} \\
 \underline{F}_{k_{uu}} &= \underline{P}_{Hk_u}^T \cdot \underline{R}_k \cdot \underline{P}_{Hk_u} + \frac{1}{2} \underline{r}_k \cdot \underline{P}_{Hk_{uu}}
 \end{aligned}
 \dots (8.15)$$

We have thus fitted an elliptic paraboloid in \underline{u}_k to performance index F_k of the system in time interval k .

Example:

Consider the simple 3-node system of Fig. 8.1. For fixed bus loads and fixed active generation at nodes 1 and 2 the optimal system status is tabulated below.

Node	Load		Generation		Voltage	
	P_D	Q_D	P_H	Q_H	E	ϕ
1	1.25	0.18	0.20	.2821	1.0237	-.1083
2	0.30	0.20	0.65	.2444	1.0488	-.0239
3	0.45	0.02	1.1828	-.0205	1.05	0

Fig. 8.3

Units of ϕ are radians. Other units are p.u. on 100 MVA base.

In the neighbourhood of this base case, we may accurately represent variations δP_{H_3} in the optimal output, P_{H_3} , of plant 3 in terms of variations δP_{H_1} and δP_{H_2} in the outputs P_{H_1} and P_{H_2} of hydro plants 1 and 2 as

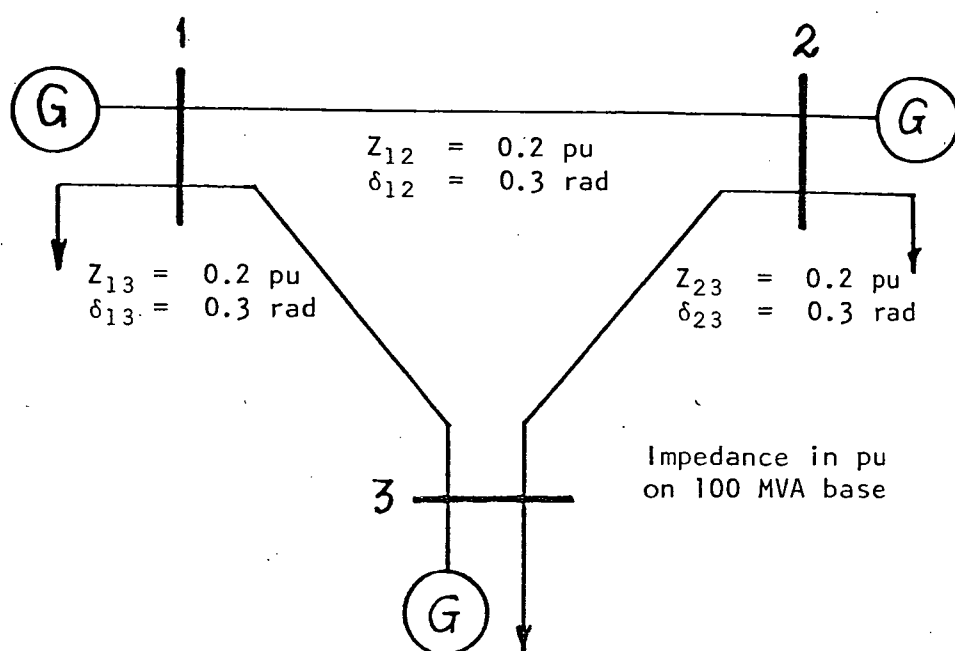


Fig. 8.1 Three node-system.
Node 3 is the slack bus

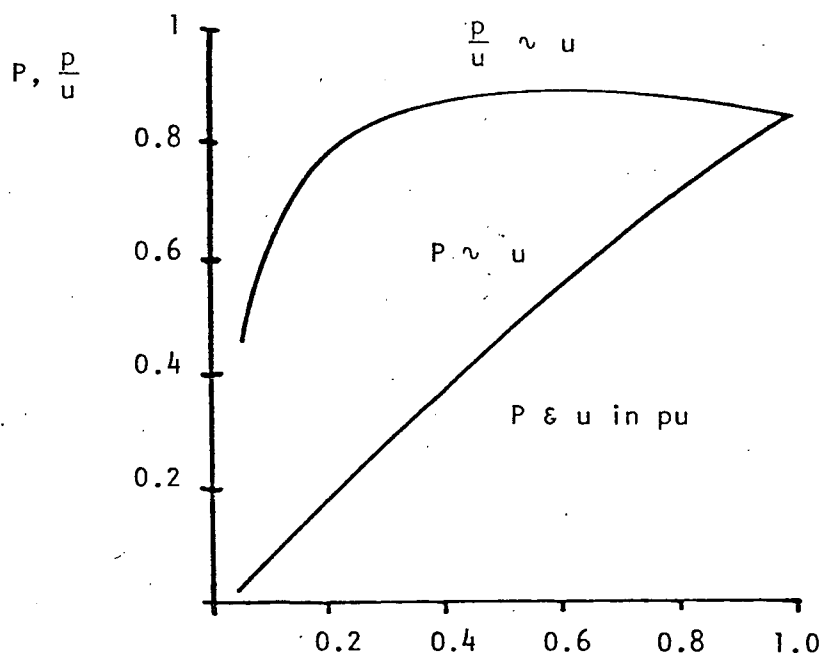


Fig. 8.2 Output and specific output curves
for hydro plants 1 & 2 of Fig. 8.1

$$\begin{aligned} \delta P_{H_3} = & .0436 \delta P_{H_1}^2 + .0365 \delta P_{H_2}^2 + .0413 \delta P_{H_1} \cdot \delta P_{H_2} \\ & - 1.06958 \delta P_{H_1} - 1.01496 \delta P_{H_2} \end{aligned}$$

... (8.16)

Plants 1 and 2 have the specific output characteristic of Fig. 8.12, and at the optimum operating point

$$\begin{aligned} \frac{\partial P_{H_1}}{\partial u_1} &= 0.95 & ; & \quad \frac{\partial P_{H_2}}{\partial u_2} = 0.87 \\ \frac{\partial^2 P_{H_1}}{\partial u_1^2} &= -0.20 & ; & \quad \frac{\partial^2 P_{H_2}}{\partial u_2^2} = -0.20 \end{aligned}$$

... (8.17)

So from equation (8.13)

$$\begin{aligned} \delta P_{H_3} = & 0.1463 \delta u_1^2 + 0.1291 \delta u_2^2 + 0.03413 \delta u_1 \cdot \delta u_2 \\ & - 1.01610 \delta u_1 - 0.88302 \delta u_2 \end{aligned}$$

... (8.18)

Consider both increases and decreases in flow rate at nodes 1 and 2 of 0.1 pu. Changes in the optimal generation at node 3 are tabulated below both as calculated by the local approximation of equation (8.19) and as calculated by a full optimal load flow computation.

Su_1	Su_2	δP_{H_3} (eqn 8.19)	δP_{H_3} (O L F)
+ 0.1	0	- .100147	- .100155
- 0.1	0	+ .103073	+ .103080
0	+ 0.1	- .087010	- .087016
0	- 0.1	+ .089593	+ .089599
+ 0.1	+ 0.1	- .186816	- .186843
- 0.1	- 0.1	+ .193007	+ .193035
+ 0.1	- 0.1	- .010896	- .010896
- 0.1	+ 0.1	+ .015722	+ .015724

Fig. 8.4 Comparison of actual and estimated optimal generation of node 3 for deviations in hydro plant flow rate at node 2.

From the results of Fig. 8.4 which represent 40% deviations in the flow rate of hydro plant 1 and 14% deviations in the flow rate of hydro plant 2, it is obvious that the representation of the optimal performance index as an elliptic paraboloid is valid, with errors of only .015% in performance index change for +40% and +14% deviations in flow rates of plants 1 and 2. The plant performance is not known to anything approaching this accuracy.

8.2 INTEGRATION OF THE HYDRO AND ELECTROTHERMAL SUBSYSTEMS

In the terminology of section 8.1, the problem may now be stated as

$$\text{minimise } \sum_{k=1}^m \delta F_k \quad \dots (8.19)$$

subject to total flow constraints

$$\sum_{k=1}^m \underline{\delta u}_k = \underline{0} \quad \dots (8.20)$$

where $\underline{\delta u}_k$ is the vector of deviations in hydro plant flow rates from those of the optimal step loading schedule (chapter 4) for time interval k .

For cascaded plants with small storage, the following storage volume constraints apply:

$$\underline{\tilde{D}}_k \leq \sum_{k=1}^{\ell} \underline{\delta u}_k - \underline{\delta u}_{k-1} \leq \hat{\underline{D}}_k \quad \dots (8.21)$$

for $\ell = 1, \dots, m$

Defining

$$f = \sum_{k=1}^m \delta F_k = \sum_{k=1}^m \underline{a}_{ok}^T \cdot \underline{\delta u}_k + \underline{\delta u}_k^T \cdot [\underline{A}_k] \cdot \underline{\delta u}_k \quad \dots (8.22)$$

$$\underline{g} = \sum_{k=1}^m \underline{\delta u}_k \quad \dots (8.23)$$

$$\underline{\hat{h}}_{\ell} = \sum_{k=1}^{\ell} (\underline{\delta u}_k - \underline{\delta u}_{k-1}) - \hat{\underline{D}}_k, \ell=1, \dots, m \quad \dots (8.24)$$

$$\underline{\tilde{h}}_{\ell} = \underline{\tilde{D}}_k - \sum_{k=1}^{\ell} (\underline{\delta u}_k - \underline{\delta u}_{k-1}), \ell=1, \dots, m \quad \dots (8.25)$$

where plant k is the first downstream neighbour of plant $k-1$,
and put

$$L = f + \underline{\lambda}^T \cdot \underline{g} + \sum_{\ell=1}^m (\underline{\hat{\mu}}_{\ell}^T \cdot \underline{\hat{h}}_{\ell} + \underline{\check{\mu}}_{\ell}^T \cdot \underline{\check{h}}_{\ell}) \quad \dots (8.26)$$

where $\underline{\lambda}$ is the column vector of Lagrange multipliers λ_i .

Ignoring the inequality constraints (8.21) on cascaded plants for the present, we put

$$L = f + \underline{\lambda}^T \cdot \underline{g} \quad \dots (8.27)$$

so at the optimum

$$\underline{L}_{\underline{u}_k} = 2 [\underline{A}_k] \cdot \underline{\delta u}_k + \underline{a}_{0k} + \underline{\lambda} = \underline{0} \quad \dots (8.28)$$

$$\underline{L}_{\underline{\lambda}} = \sum_{k=1}^m \underline{\delta u}_k = \underline{0} \quad \dots (8.29)$$

Rearranging (8.28)

$$\underline{2u}_k + [\underline{A}_k]^{-1} \cdot (\underline{a}_{0k} + \underline{\lambda}) = \underline{0} \quad \dots (8.30)$$

and summing (8.30) over time intervals $k=1, \dots, m$

$$2 \sum_{k=1}^m \underline{u}_k + \sum_{k=1}^m [\underline{A}_k]^{-1} \cdot (\underline{a}_{0k} + \underline{\lambda}) = \underline{0} \quad \dots (8.31)$$

substituting (8.29) into (8.31) and rearranging,

$$\underline{\lambda} = - \left(\sum_{k=1}^m [\underline{A}_k]^{-1} \right)^{-1} \cdot \left(\sum_{k=1}^m [\underline{A}_k]^{-1} \cdot \underline{a}_{0k} \right) \quad \dots (8.32)$$

The improved schedule for the system is obtained by substituting $\underline{\lambda}$ into each of (8.30). This process, requiring inversion of $m+1$

matrices of dimension $(n_H - 1)$ at each iteration, together with formation of these matrices, is cumbersome if the number of hydro plants, n_H , is large.

A simplification, replacing the matrix $[A_k]$ by its diagonal, is introduced in 8.3 and shown to result in a marked decrease in computation per iteration, although at the expense of the number of iterations required for convergence.

8.2.1 A Simplification to Improve Computing Efficiency

The system performance index is a measure of efficiency of operation of the power system, and for fixed system loads, a change in the performance index reflects a change in system losses.

If the instantaneous performance index of an all hydro system is the power input of the slack bus plant,

$$\text{i.e. } F = k_n \cdot u_n \quad \dots (8.33)$$

then at any instant, since system loads are fixed,

$$\begin{aligned} \sum_{i=1}^n k_i \cdot \delta u_i &= \delta(\text{system losses}) \\ &= \delta(\text{plant losses}) + \delta(\text{transmission losses}) \\ &\dots (8.34) \end{aligned}$$

where $k_i \cdot u_i$ is the power input of hydro plant i proportional to the flow rate u_i of that plant. The performance index is therefore

$$\begin{aligned} F = k_n \cdot u_n &= \delta(\text{plant losses}) \\ &+ \delta(\text{transmission losses}) \\ &- \delta(\text{total energy input to base load plants}) \\ &\dots (8.35) \end{aligned}$$

In expression (8.35),

$$\text{plant losses} = \sum_{i=1}^n (k_i u_i - P_{Hi}) \quad \dots (8.36)$$

total energy input to base load plants

$$= \sum_{i=1}^{n-1} k_i u_i \quad \dots (8.37)$$

and for illustration purposes only

$$\text{transmission losses} \approx \sum_{i=1}^n \sum_{j=1}^n P_{Hi} \cdot B_{ij} \cdot P_{Hj} + \sum_{i=1}^n B_{0i} \cdot P_{Hi} + B_{00} \quad \dots (8.38)$$

where B_{ij} , B_{0i} and B_{00} are the traditional B-coefficients.^[2,3,4] Neither (8.36) nor (8.37) contain off-diagonal products between plant discharge rates or power outputs. These terms appear only in the transmission losses, where, depending on the degree of inter-connection of the power system, they are not dominant.

At the possible expense of increasing the number of iterations required for convergence, we can omit the off-diagonal product terms in the fitted paraboloid, equations (8.28) and (8.32) reducing to

$$L_{u_{ik}} = 2A_{iik} \cdot u_{ik} + a_{0ik} + \lambda_i = 0 \quad \dots (8.39)$$

and

$$\lambda_i = - \left(2 \sum_{k=1}^m u_{ik} + \sum_{k=1}^m \frac{a_{0ik}}{A_{iik}} \right) / \sum_{k=1}^m \frac{1}{A_{iik}} \quad \dots (8.40)$$

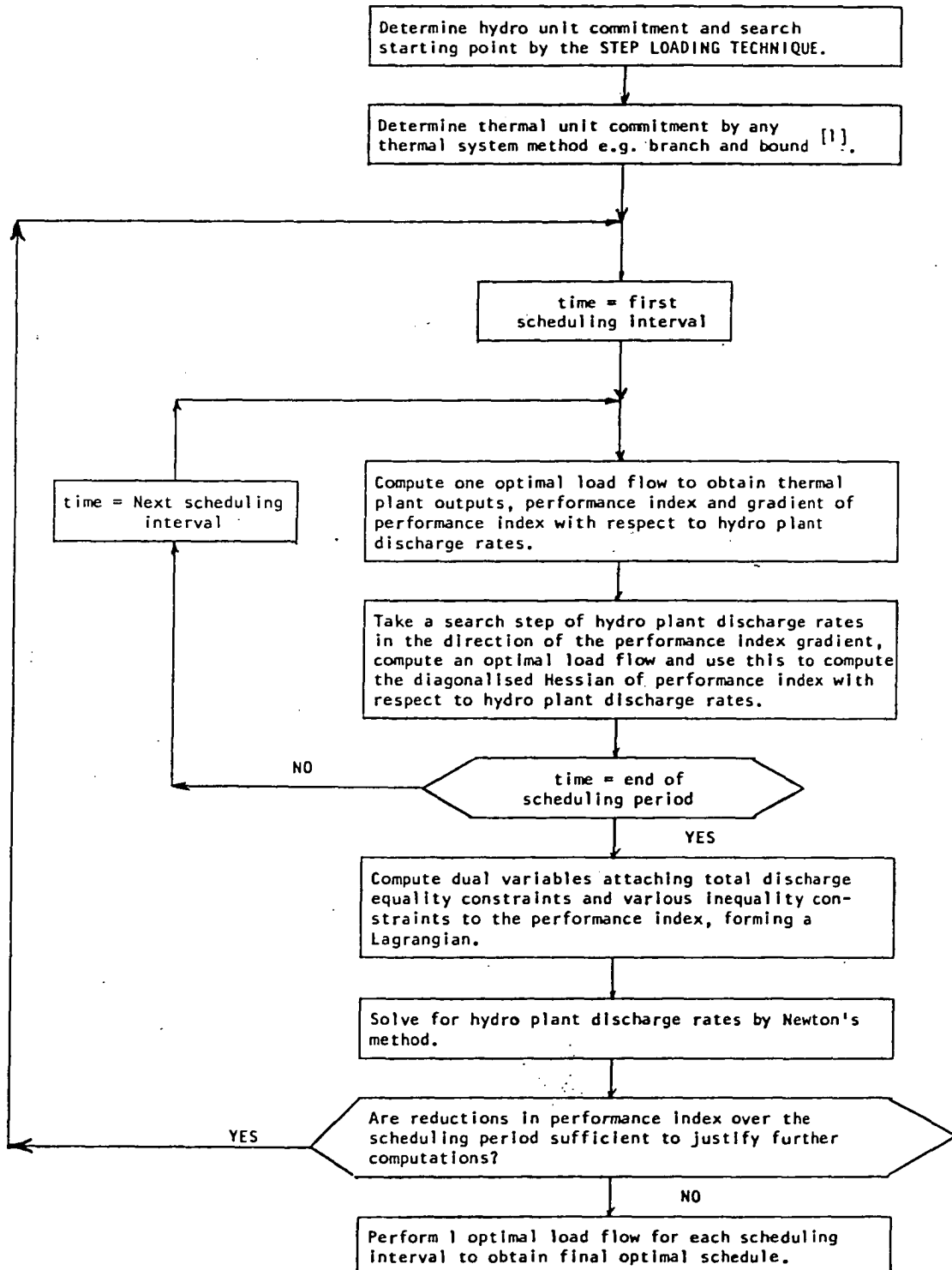


Fig. 8.5 Flow Chart of the scheme for hydrothermal system scheduling

respectively for $i = 1, 2, \dots, n_H - 1$ and $k = 1, 2, \dots, m$.

The need to invert an $n_H - 1$ square, non-sparse matrix at each iterative step is therefore avoided, and the quadratic coefficients A_{iik} and a_{0ik} may be found by a simple exploratory search step rather than directly calculated as this involves less computing. The flowchart for this simplified scheme is shown in Fig. 8.5.

This is the form of the general second derivative hill-climbers discussed in section 5.3.2 and evaluated by Monte-Carlo tests on a simulated scheduling of a four plant hydro system.

8.3 EXAMPLE PROBLEM ILLUSTRATING SCHEDULING OF THE INTEGRATED SYSTEM

In considering examples to illustrate scheduling of the integrated power system we must include the step-loading techniques of chapter 4, as the optimal step loading schedule is used as the starting point for a full integrated schedule.

The step loading techniques of chapter 4 were built around a quadratic approximation to system transmission losses. The traditional B-coefficient loss formula [2,3,4] is based on current injection into system nodes and assumes node voltages are unchanging. In a system under optimal control, this assumption is not valid. Section 8.1 of this chapter shows that a quadratic may be accurately fitted to system characteristics under the assumption that node voltages, instead of being fixed, are optimally controlled to minimise any predetermined objective function.

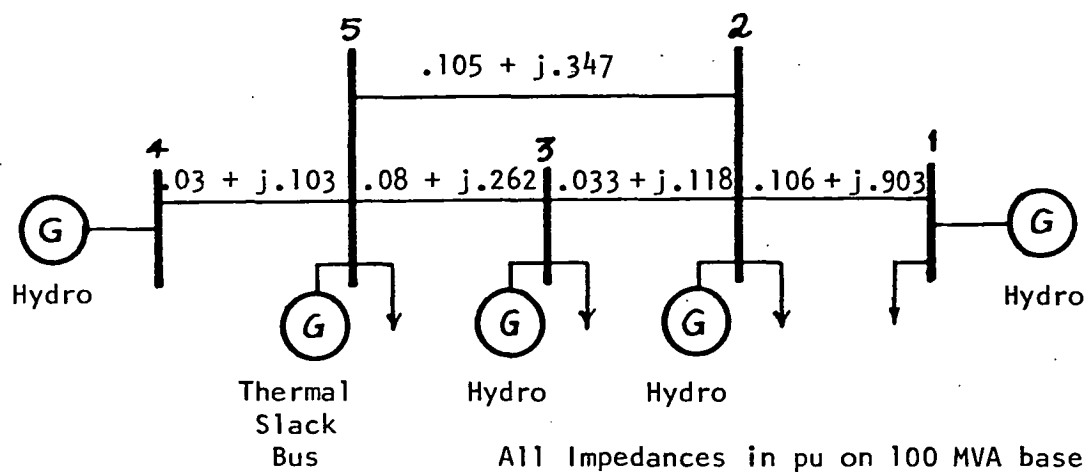


Fig. 8.6 5 Node Sample System

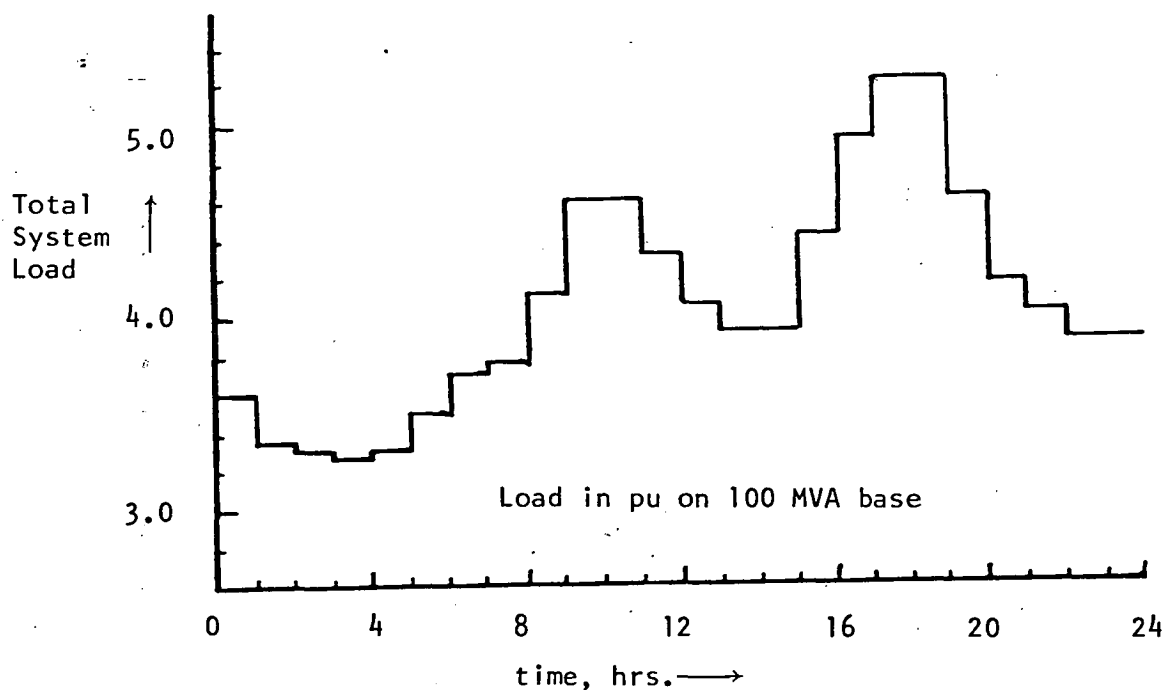


Fig. 8.7 Total System Load

Bus 5 load = 60 MW constant
 Bus 1 load = 20% of conforming load
 Bus 2 load = 40% of conforming load
 Bus 3 load = 40% of conforming load
 Bus 4 load = 0
 All loads at .895 pf

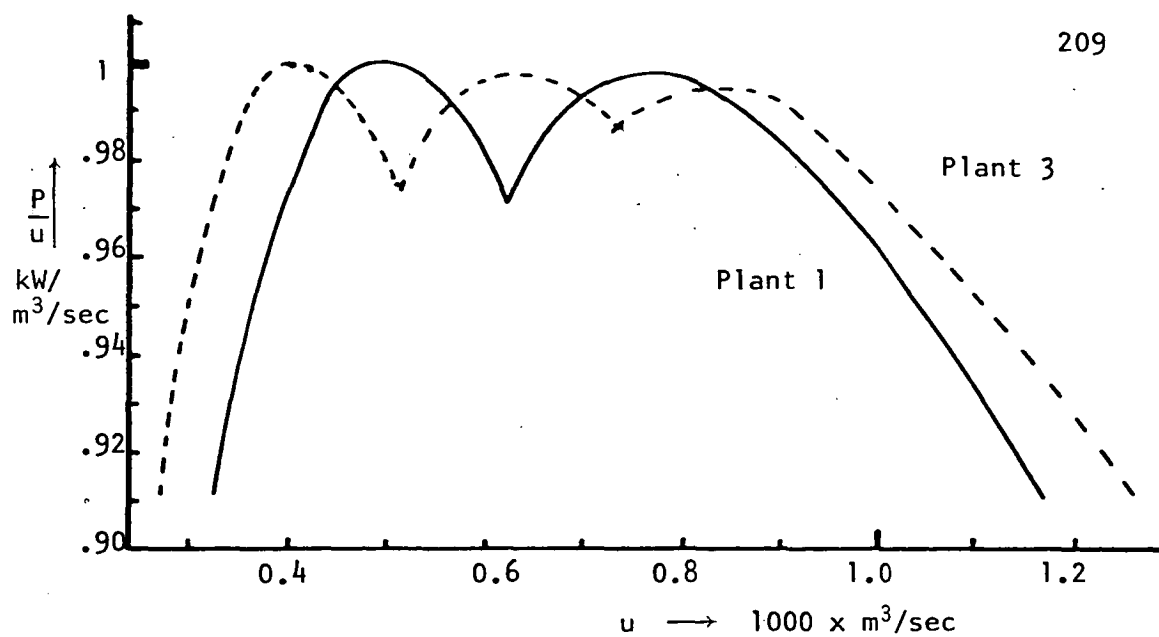


Fig. 8.8 Characteristics of hydro plants at buses 1 and 3

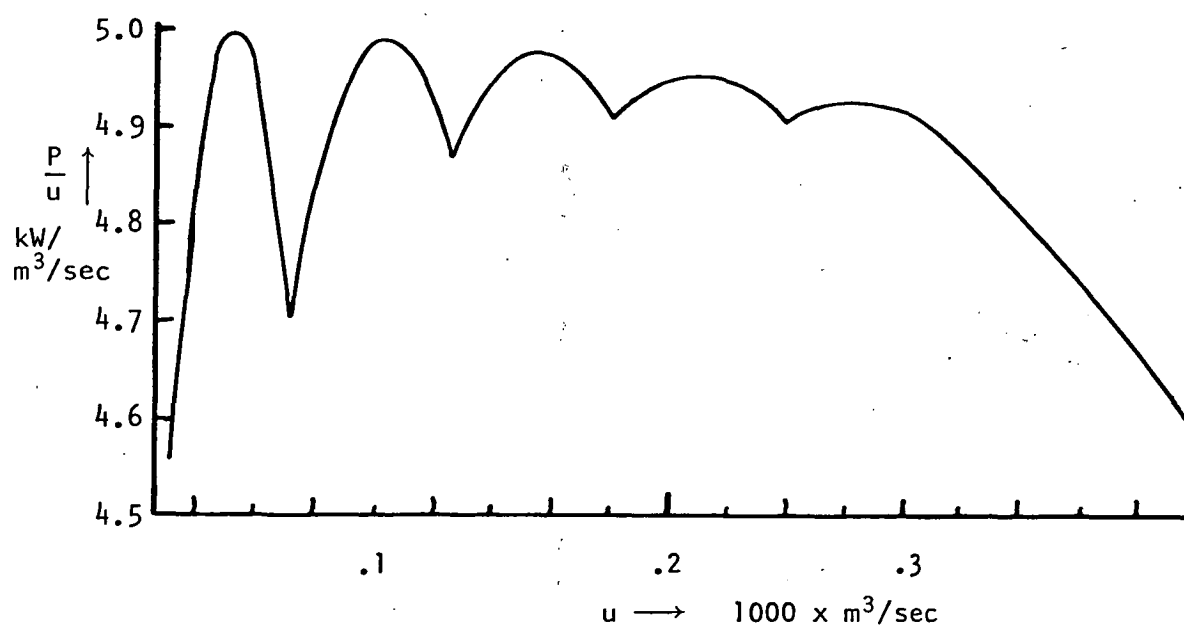


Fig. 8.9 Characteristics of hydro plant at bus 2

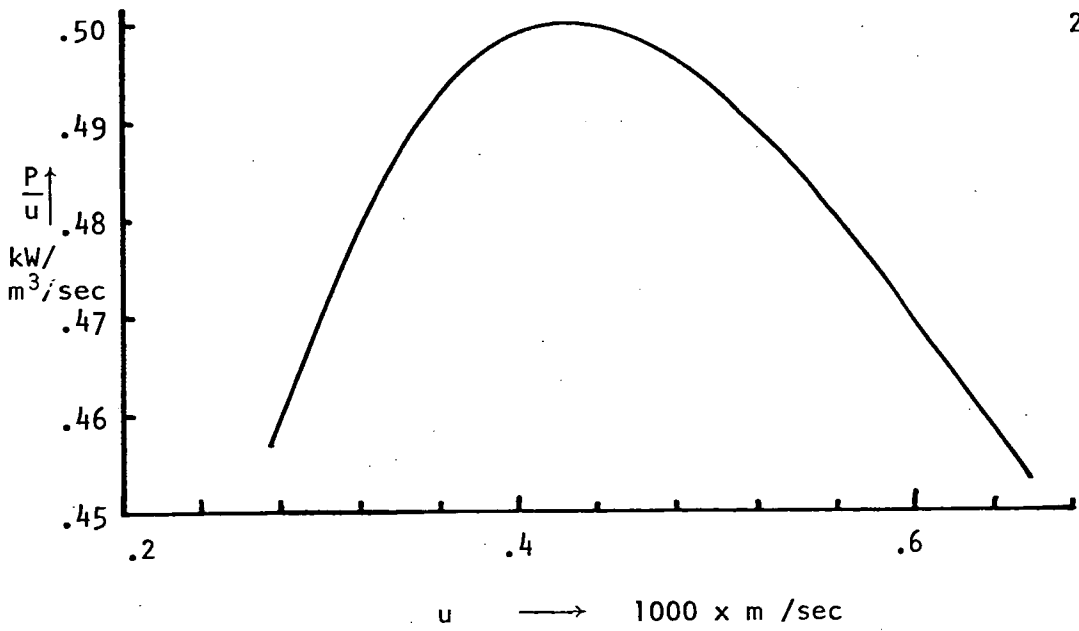


Fig. 8.10 Characteristics of hydro plant at bus 4

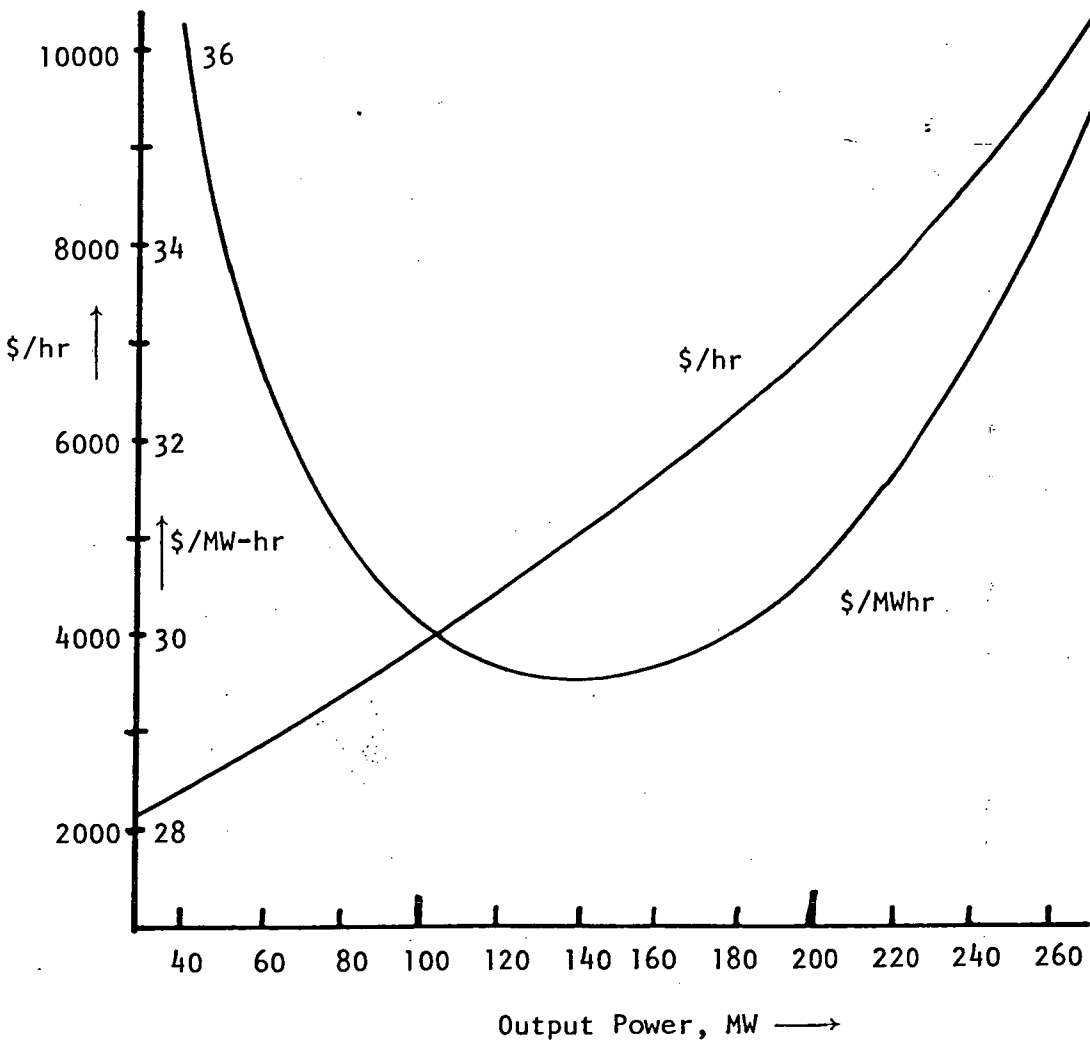


Fig. 8.11 Characteristics of thermal plant at bus 5

This alternative quadratic loss formulation is more appropriate to computation of the optimal step loading schedule and has been used in step-loading for the examples of this section, with the quadratic loss formula centred around average power outputs from all plants.

Consider the five plant system of Fig. 8.6 having bus loads over a 24 hour period as shown in Fig. 8.7 and plant characteristics as shown in Figs. 8.8 - 8.11. This system will be first scheduled by step loading to determine unit commitment, then this schedule will be improved to the optimum using optimal load flow computations and a Newton's method search.

8.3.1 Step Loading to Determine Unit Commitment

The five plant system of Fig. 8.6 comprises four hydro plants at buses 1, 2, 3 and 4 having 2, 5, 3 and 1 units each respectively, and single unit thermal plant at the slack bus, bus 5. Step modes for the four hydro plants are as tabulated below.

Plant	High State		Low State		Time in High State
	Units	MW	Units	MW	
1	2	76	1	49.5	16 hrs.
2	5	128.5	4	102.7	16 hrs.
3	3	82	2	63	8 hrs.
4	1	21	0	0	20 hrs.

Fig. 8.12 Peak efficiency step modes of the hydro plants

As shown in section 8.1 of this chapter, optimal operation of the electrothermal subsystem of the hydrothermal power system may be accurately represented by a quadratic of changes in power injection into each bus. This representation is far more appropriate to the step-loading problem than the traditional B-coefficient loss formula [2, 3, 4] since it is based on a more relevant set of assumptions.

For the 5 plant system of Fig. 8.6, the base case optimal load flow solution, based on average bus loads and generations is

Bus	Generation		Load		Voltage	
	P_G	Q_G	P_D	Q_D	E	ϕ
1	.70	.3499	.70	.35	1.0214	-.12101
2	1.20	.6532	1.40	.70	1.0215	-.12102
3	.70	.6217	1.40	.70	1.0165	-.13909
4	.20	.6963	0	0	1.0559	.01848
5	1.3382	.6027	.60	.60	1.05	0

Fig. 8.13 Base case optimal load flow

E, P, Q in pu on 100 MVA base

ϕ in radians

Optimum operation of the electrothermal subsystem can then be represented by

$$\Delta P_5 = \begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \Delta P_3 \\ \Delta P_4 \end{bmatrix}^T \begin{bmatrix} .17040 & .12175 & .09745 & 0 \\ 0 & .06085 & .09005 & 0 \\ 0 & 0 & .05880 & 0 \\ 0 & 0 & 0 & .02635 \end{bmatrix} \begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \Delta P_3 \\ \Delta P_4 \end{bmatrix}$$

$$- \begin{bmatrix} 1.07784 \\ 1.07742 \\ 1.08866 \\ 0.98931 \end{bmatrix}^T \begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \Delta P_3 \\ \Delta P_4 \end{bmatrix}$$

This representation has an error of only .0033 pu in the optimal value of bus 5 generation for an increase of .2 pu in the generation at all other buses.

Fig. 8.14 shows the optimal step loading schedule for the five plant system. The total cost of fuel used by plant 5 in the 24 hour period with plants 1-4 step loaded and the electrothermal system represented by the quadratic approximation was \$94,999. Minimum and maximum generation of plant 5 was 110.96 MW and 227.18 MW respectively.

8.3.2 Refinement of the Step Loading Schedule Using a Decomposition Technique

In addition to the information given in Figs. 8.6 - 8.14, the following constraints are in force:

- (a) Voltages at buses supply loads must be in the range 0.95 to 1.05 pu. Voltages elsewhere must be within 0.90 to 1.10 pu.
- (b) Each plant has power and VA restrictions.

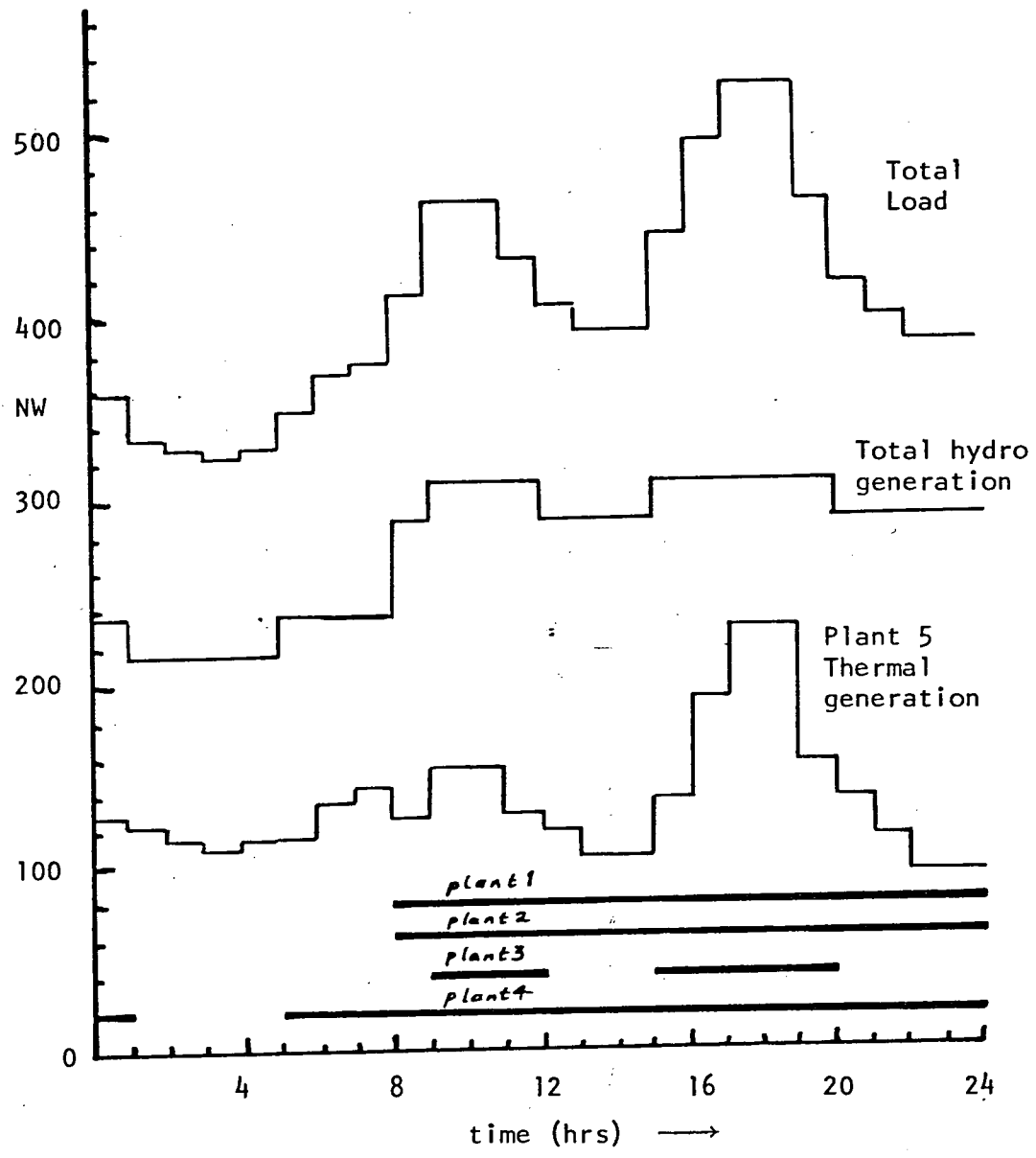


Fig. 8.14 Optimal step-loading schedule of the five plant system

Solid bars indicate plant output in the high state

Plant	Units Running	Max. MW	Max. MVA
1	1	71	88
	2	109	136
2	4	147	184
	5	184	230
3	2	90	113
	3	117	147
4	0	0	0
	1	30	37.5
5	1	270	300

Fig. 8.15 Plant power and MVA limits

Starting from the steploading schedule of Fig. 8.14, and maintaining unit commitment as determined by step-loading, a Newton's method search as detailed in the flow chart of Fig. 8.5, using the search algorithm of equations (5.21) and (5.22) was carried out. The optimal schedule is detailed in Figures (8.16) - (8.21). The total cost of fuel used by plant 5 for the this optimal schedule was \$93,627, a saving of \$1,372, or 1.44% on that used by plant 5 in the step-loading schedule. Minimum and maximum generation of plant 5 was 119.51 MW and 175.28 MW respectively.

The Newton search algorithm converged to within acceptable accuracy in 3 steps, and had converged to within computer round-off error in 8 steps. This convergence is documented in Fig. 8.22.

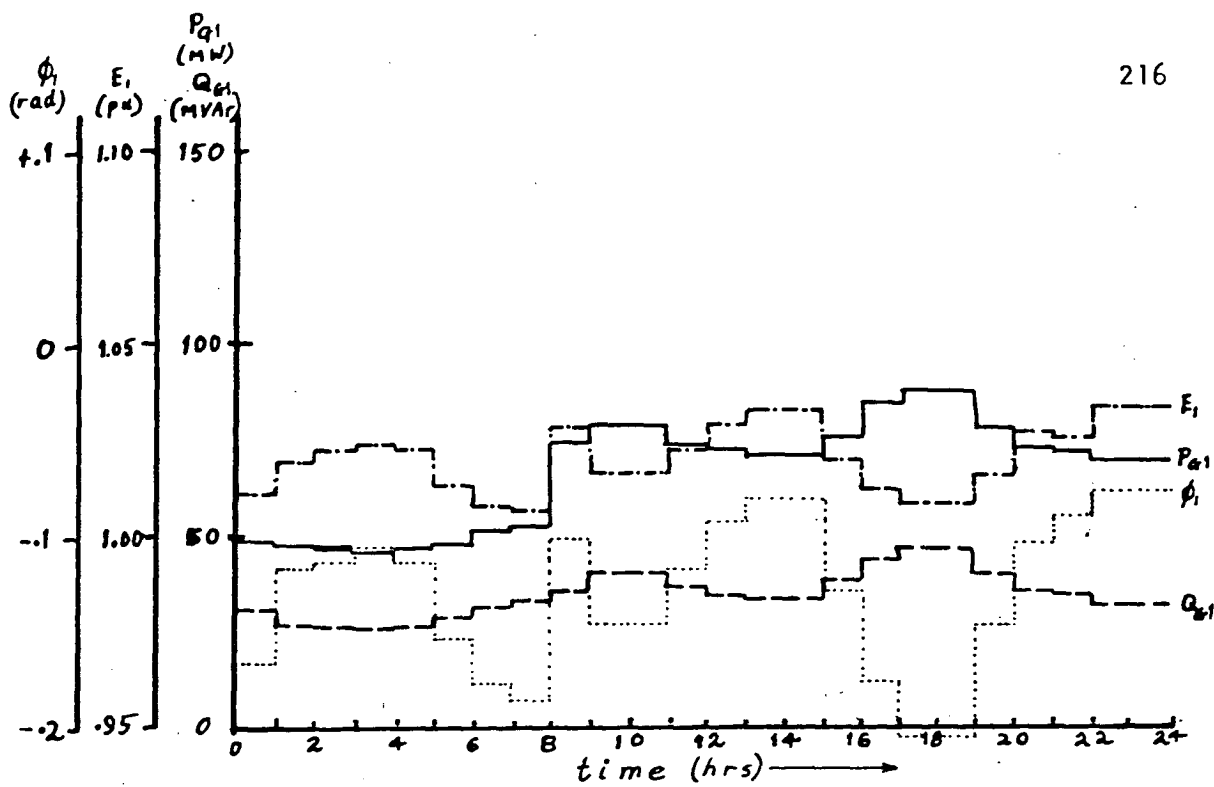


Fig. 8.16 Optimum Schedule - Bus 1 Variables

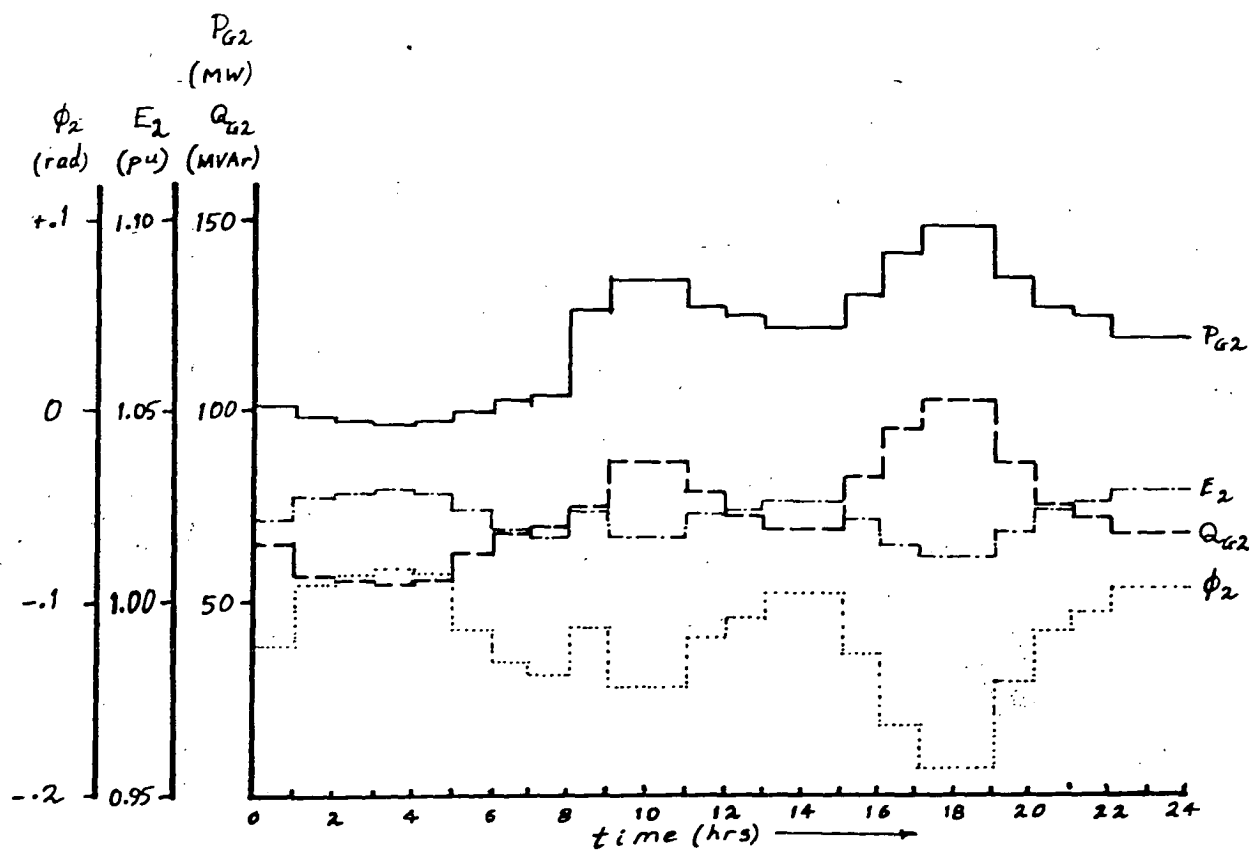


Fig. 8.17 Optimum Schedule - Bus 2 Variables

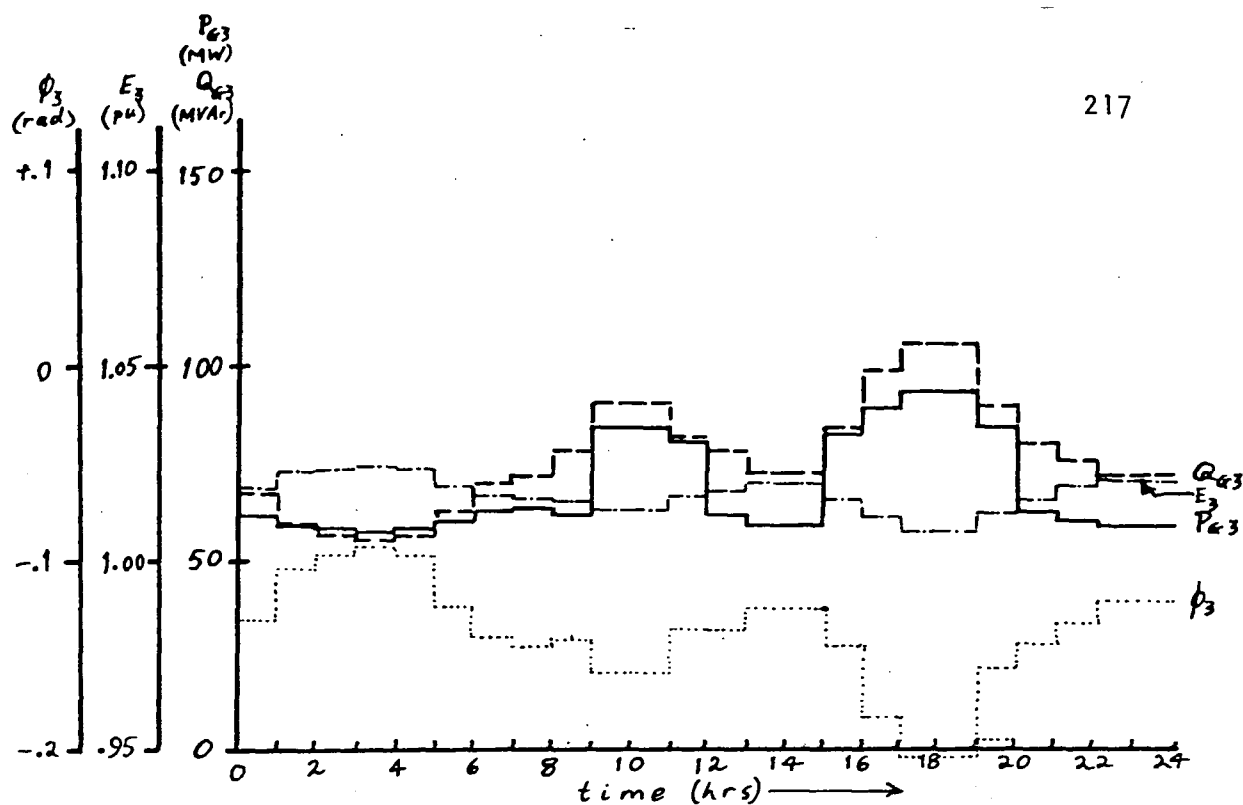


Fig. 8.18 Optimum Schedule - Bus 3 Variables

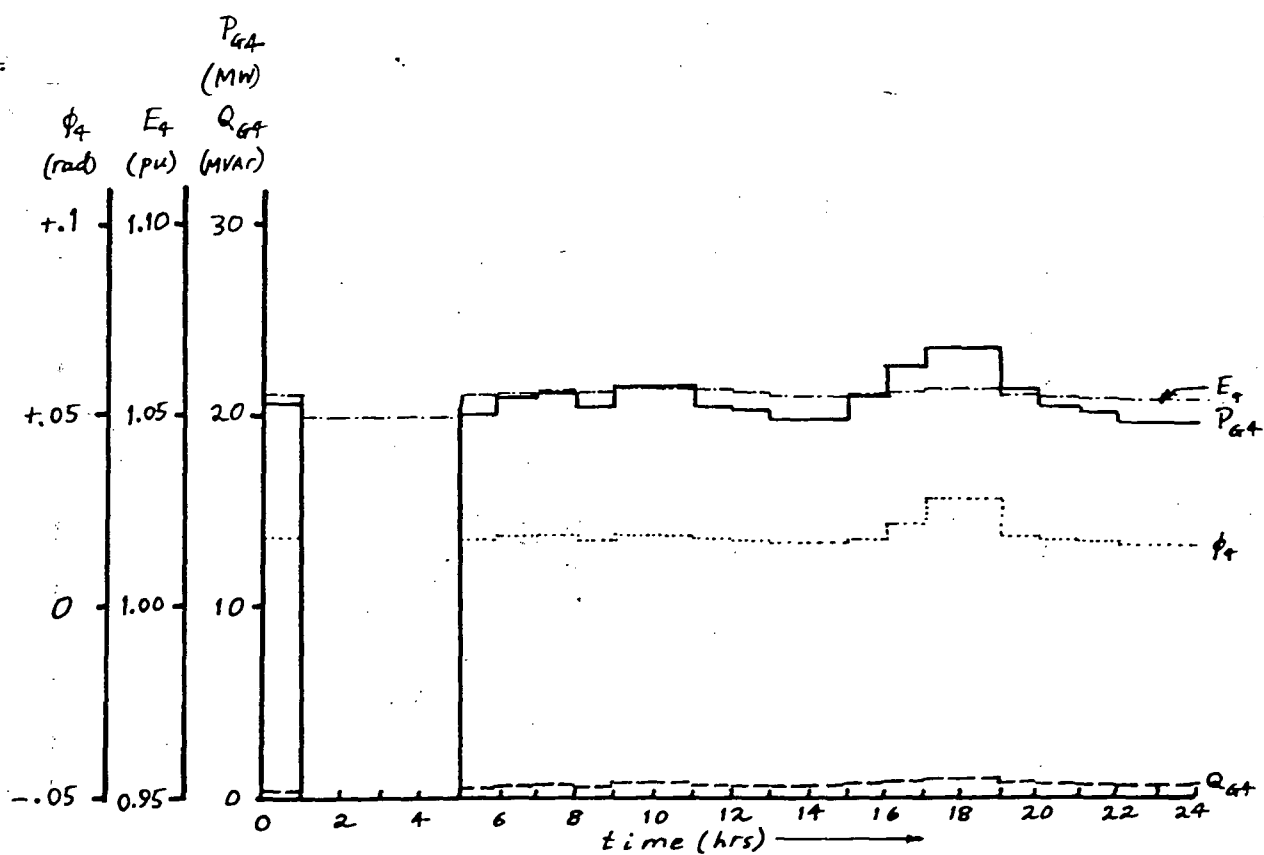


Fig. 8.19 Optimum Schedule - Bus 4 Variables

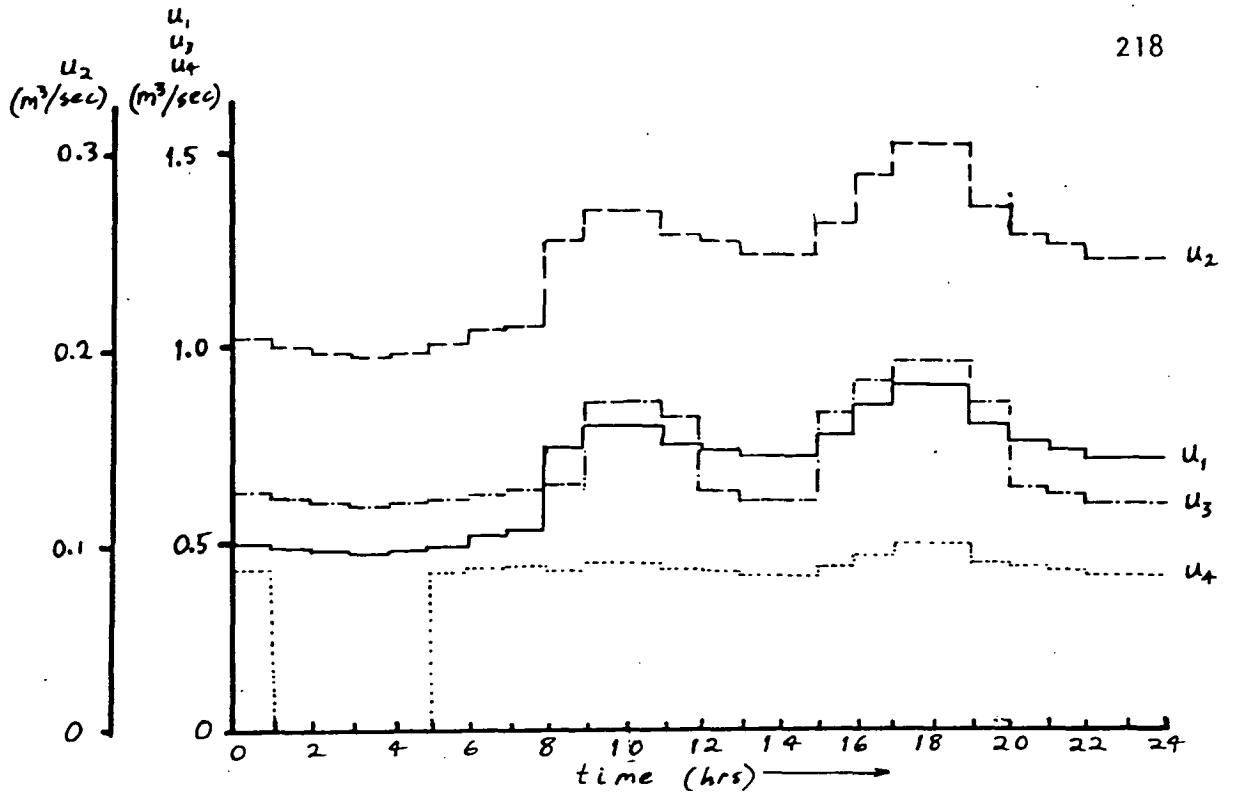


Fig. 8.20 Optimum Schedule - Hydro discharge rates

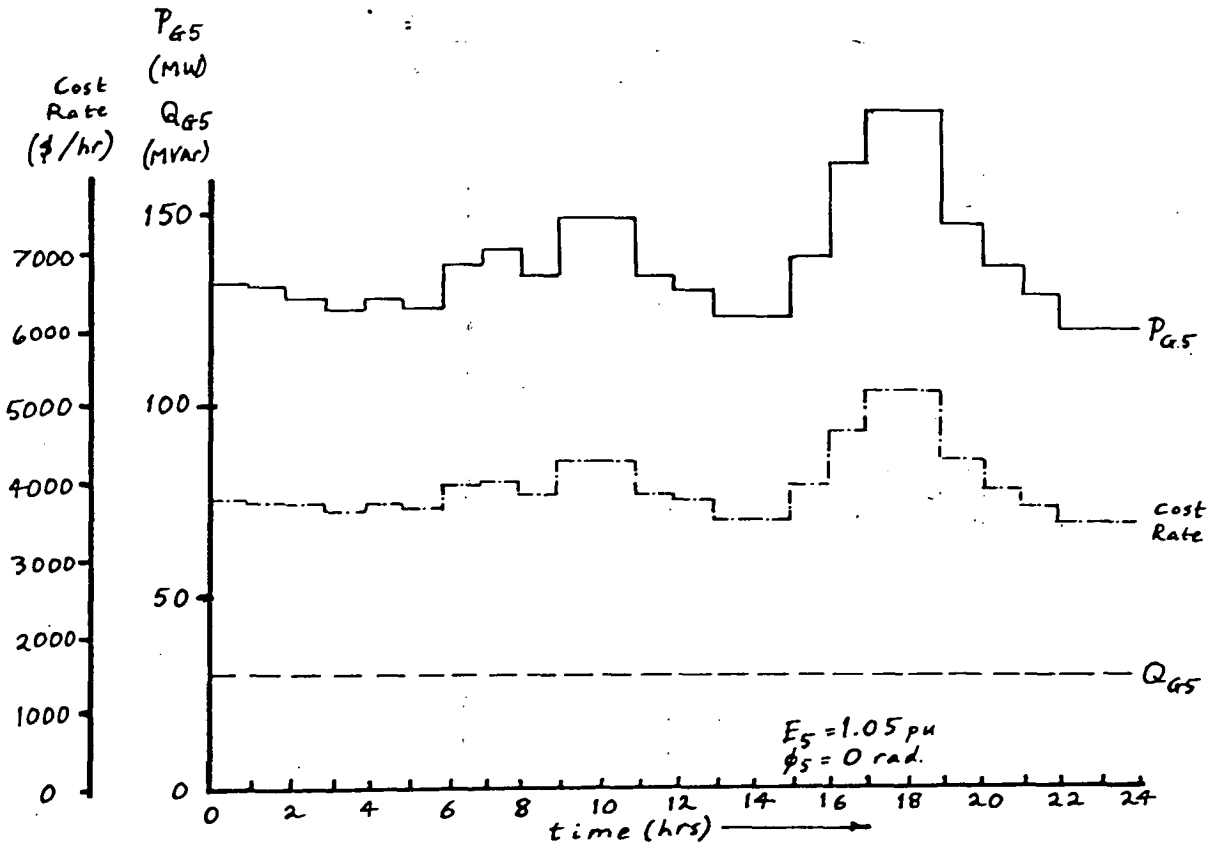


Fig. 8.21 Optimum Schedule - Bus 5 (Thermal plant) Variables

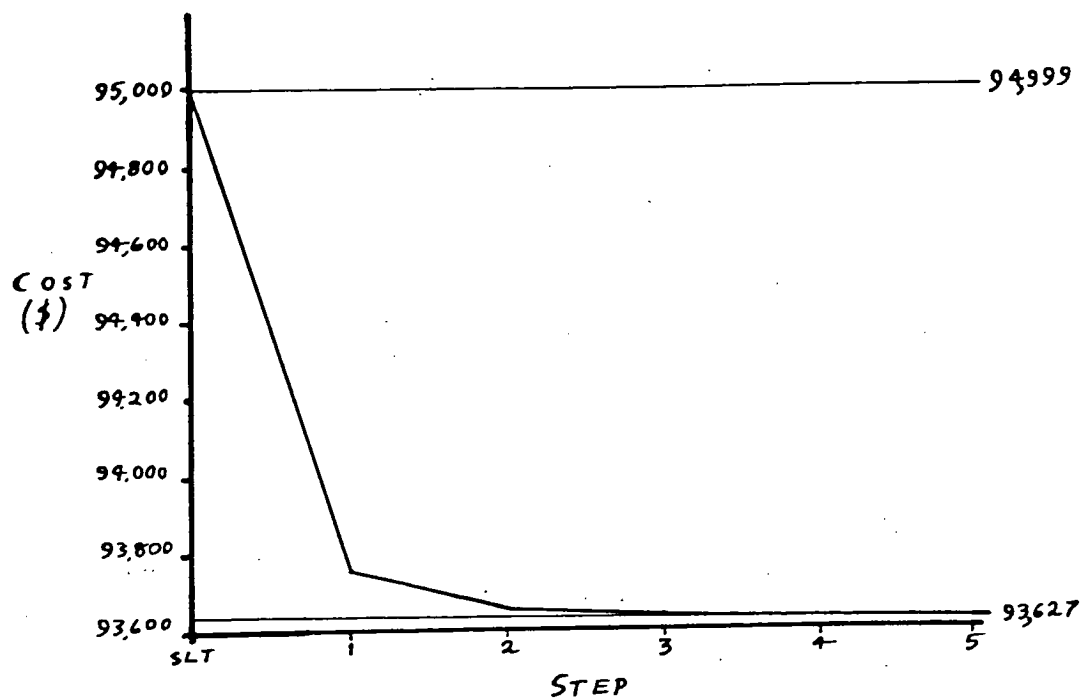


Fig. 8.22. Newton Search for Optimum
- Convergence characteristics

It is useful to note that the thermal plant output has relatively little variation except at times of peak load. This is in keeping with the assumptions made in section 4.3 to justify step-loading based on the load-duration curve, and can be explained by the fact that a sharp increase in thermal power at one time and a dip at another time gives rise to a large cost rise and a lesser cost decrease respectively as the incremental cost of thermal power increases significantly with output. The smaller the fraction of total system capacity met by the thermal plant, the more pronounced this levelling process will become.

8.4 DISCUSSION

The optimisation strategy introduced in chapter five for the hydrothermal power system was detailed in this chapter. The method utilised decomposition of the hydrothermal power system into a hydro subsystem and an electrothermal subsystem. In solving the hydro subsystem, the variational problem was replaced by a series of static problems corresponding to a discrete set of time intervals, together with overall total discharge constraints.

At each iteration, the schedule of the electrothermal subsystem was prepared using a set of optimal load flow solutions, one for each time interval, incorporating system transmission losses and electrothermal subsystem constraints in their most accurate form. Sensitivity information generated by the optimal load flow solutions was then used to provide a set of second order approximations to optimal performance of the electrothermal subsystem including equality and inequality constraints, which were then used to improve the hydro subsystem schedule by a Newtons method search.

By virtue of its modular approach, this method is able to incorporate any future developments in the optimal load flow field which is still being actively researched. The use of existing optimal load flow algorithms for electrothermal scheduling and a Newton search for the hydro schedule make a great deal of sensitivity information readily available. This is explained in the next chapter.

8.5 REFERENCES

- [1] Dillon, T.S. "Problems of optimal economic operation and control of integrated (hydrothermal) and thermal power systems". Ph.D. thesis, Department of Electrical Engineering, Monash University (1974)
- [2] Early, E.D., Watson, R.E. "A new method of determining constants for the general transmission loss equation". Trans. AIEE Pwr. Appar. Sys. v. 74 (1956) pp 1417-1423
- [3] Early, E.D., Watson, R.E., Smith G.L. "A general transmission loss equation". Trans. AIEE Pwr. Appar. Sys. v. 74 (1955) pp 510-520
- [4] George, E.E. "Intrasystem transmission losses". Trans. AIEE v. 62 (1944) pp 153-158

9. SENSITIVITY CONSIDERATIONS IN A HYDROTHERMAL POWER SYSTEM

Effective implementation of an optimal scheduling method requires that we determine the accuracy of modelling and measurement necessary in the power system and river system. If excessive, the cost of obtaining extra accuracy is not reflected in a significant reduction in operating costs. Conversely, inadequate accuracy results in high operating costs as the scheduled operating point is not a true optimum. The most appropriate accuracy is therefore a compromise between the direct cost of achieving accuracy and the loss of economy resulting from inaccuracy. This compromise is resolved by examining the loss of economy due to a perturbation in the system model or in measured or predicted parameters. Specifically, a sensitivity analysis of the optimal schedule is needed.

Such a sensitivity analysis identifies parameters with most influence on the schedule. It also quantifies deterioration in system performance due to poor system modelling or state estimation and measurement.

The optimal schedule of a hydrothermal power system is computed in advance because of the variational nature of the problem. It must therefore be based on predictions of system loads and stream flows up to at least 24 hours ahead. The ability to produce a slightly altered schedule corresponding to a small demand or stream flow variation without the large number of load flow computations involved in the initial solution is desirable, and such a method may be derived from second order sensitivity considerations.

Sensitivity information may also be utilised to minimise the frequency of scheduling intervals in the initial system schedule and of corrections made to the schedule in the light of demand and stream flow variations.

Other Information supplied by a sensitivity analysis is useful in system planning, particularly in highlighting and costing various system limitations.

The role of sensitivity analysis as a guide to operators in a system without on-line rescheduling is important, supplying the basic criteria for modification of schedules to suit changed loads and river flows, allowing economic generation after a minor change in system configuration such as that resulting from plant failure. Based on the work of Peschon, Piercy, Tinney and Tsviet [7] an almost complete first order sensitivity analysis of the electrothermal subsystem may be obtained utilising information calculated in the optimal power flows used to schedule the subsystem. Peschon et al. extended this to second order but excluded inequality constraints from the second order analysis. An extension of the sensitivity analysis to allow rescheduling in the presence of inequality constraints was achieved by Dillon, Morsztyn and Tun [4], who also developed a method for determination of scheduling intervals for a thermal system. References [8] & [9] have developed an extension of the sensitivity analysis method for optimal control problems to systems with state dependent control constraints and state variable constraints, and then carried an application of this method to the hydro-thermal scheduling problem.

The hydro subsystem scheduling method presented in this thesis may be based on a Newton's method search, and, as a result of this, some sensitivity information is directly available. This, together with the basic quadratic approximation concept used in

- .. coordination of hydro and electrothermal schedules form a suitable basis for a simple schedule updating procedure.

9.1 THE ELECTROTHERMAL SUBSYSTEM

Sensitivity relations in thermal power system scheduling have been explored by several researchers [3, 4, 7]. Such relations may be applied without modification to the electrothermal subsystems of integrated power systems.

There are two major categories of sensitivity relations in the electrothermal subsystem, sensitivity of one electrical variable to another and sensitivity of the performance index, frequently cost, to changes in the electrical variables, primarily active and reactive demands at each node or active generation at hydro power nodes. The sensitivity of the performance index to relaxation of electrical system constraints, such as those on node voltage or line flows is also important, together with sensitivity to the frequency of rescheduling, although results obtained by analysis of the electrothermal system alone [3, 4] are not directly applicable to the variational problems posed by an integrated power system.

9.1.1 Sensitivity Relations Between Electrical Variables

The power flow equations for a system with off-nominal transformers and phase shifters are (3.3) and (3.4) in chapter 3. These may be expressed in control language as

$$\underline{g}(\underline{x}, \underline{u}, \underline{p}) = \underline{0} \quad \dots (9.1)$$

where \underline{u} , \underline{x} and \underline{p} are the imposed or control variables,

dependent or state variables and uncontrollable or parameter variables respectively as explained in chapter 7.

Equation (9.1) may be efficiently solved by Newton's method:

1. Expand (9.1) as a Taylor series about an assumed solution \underline{x}^0 and write

$$\underline{g}(\underline{x}^0, \underline{u}, \underline{p}) + [\underline{g}_x]^T \cdot \underline{\delta x} = \underline{0} \quad \dots (9.2)$$

where $[\underline{g}_x]$ is the Jacobian matrix of partial derivatives $\frac{\partial g_i}{\partial x_i}$ evaluated at assumed solution \underline{x}^0

2. Calculate the correction

$$\underline{\delta x} = - [\underline{g}_x]^{-1} \cdot \underline{g} \quad \dots (9.3)$$

3. Repeat with new assumed solution

$$\underline{x}^1 = \underline{x}^0 + \underline{\delta x} \quad \dots (9.4)$$

until correction $\underline{\delta x}$ is sufficiently small.

The calculation $[\underline{g}_x]^{-1}$ is commonly carried out by optimally ordered triangular factorisation and sparsity programming. [8]

If a solution, \underline{x}^* , of the load flow equations, corresponding to nominal control and parameter vectors \underline{u}^* and \underline{p}^* has been found,

$$\underline{g}(\underline{x}^*, \underline{u}^*, \underline{p}^*) = \underline{0} \quad \dots (9.5)$$

To derive the sensitivity relations, we determine changes $\underline{\delta x}$ resulting from small changes $\underline{\delta p}$ and $\underline{\delta u}$ away from the nominal \underline{p}^* and \underline{u}^* . Expanding the perturbed load flow equations to first

order in a Taylor series gives

$$\begin{aligned} \underline{g}(\underline{x}^* + \underline{\delta x}, \underline{u}^* + \underline{\delta u}, \underline{p}^* + \underline{\delta p}) &= \underline{0} \\ &\approx g(\underline{x}^*, \underline{u}^*, \underline{p}^*) + [g_x] \cdot \underline{\delta x} + [g_u] \cdot \underline{\delta u} + [g_p] \cdot \underline{\delta p} \\ &\dots (9.6) \end{aligned}$$

so from (9.5) and (9.6)

$$[g_x] \cdot \underline{\delta x} + [g_u] \cdot \underline{\delta u} + [g_p] \cdot \underline{\delta p} \approx \underline{0} \dots (9.7)$$

$$\underline{\delta x} \approx -[g_x]^{-1} \cdot [g_u] \cdot \underline{\delta u} - [g_x]^{-1} \cdot [g_p] \cdot \underline{\delta p} \dots (9.8)$$

with $[g_x]$, $[g_u]$ and $[g_p]$ evaluated at $(\underline{x}^*, \underline{u}^*, \underline{p}^*)$

Expressing (9.8) in terms of sensitivity matrices

$$\underline{\delta x} = \begin{bmatrix} S_x^u \\ S_x^p \end{bmatrix} \cdot \underline{\delta u} + \begin{bmatrix} S_p^x \end{bmatrix} \cdot \underline{\delta p} \dots (9.9)$$

$$\text{i.e.} \quad \begin{bmatrix} S_u^x \end{bmatrix} = -[g_x]^{-1} \cdot [g_u] \dots (9.10)$$

$$\begin{bmatrix} S_p^x \end{bmatrix} = -[g_x]^{-1} \cdot [g_p] \dots (9.11)$$

While evaluation of matrices $[g_x]$, $[g_u]$ and $[g_p]$ is trivial, inversion of $[g_x]$ is not, as it may have a dimension of up to twice the number of system nodes. This inverse is available in factored form from the Newton's method power flow used to obtain solution \underline{x}^* .

The foregoing analysis is that formulated by Peschon, Piercy, Tinney and Tviet [7] who considered it to be an extension of reference [8]. In their formulation, they considered only fixed ratio transformers and phase shifters. If transformers carrying out automatic regulation of node voltages and phase shifters carrying out automatic regulation of line flows are handled within the Newton algorithm as detailed in chapter 7, the factored inverse

$[g_x]^{-1}$ obtained from a load flow analysis will include these, and their respective sensitivity relations may be obtained with no additional calculation.

9.1.2 Sensitivity of the Objective Function to the Electrical Variables

The well known gradient technique for optimal instantaneous operation of a power system, first proposed by Tinney and Hart [8] is described in detail in chapter 7. In the control language of 9.1.1, if $F(\underline{x}, \underline{u})$ is the performance index used to assess system operation, the problem may be expressed as:

$$\text{Min}_{\underline{u}} F(\underline{x}, \underline{u}) \quad \dots (9.12)$$

subject to

$$\underline{g}(\underline{x}, \underline{u}, \underline{p}) = \underline{0} \quad \dots (9.13)$$

and

$$\underline{\check{x}} \leq \underline{x} \leq \underline{\hat{x}} \quad \dots (9.14)$$

$$\underline{\check{u}} \leq \underline{u} \leq \underline{\hat{u}} \quad \dots (9.15)$$

The Kuhn-Tucker theorem provides a set of necessary conditions acting on \underline{x} and \underline{u} at the minimum of F . These are normally also sufficient. A Lagrangian $L(\underline{x}, \underline{u}, \underline{p}, \underline{\lambda}, \underline{\mu}, \underline{v})$ may be defined as

$$\begin{aligned} L = F(\underline{x}, \underline{u}) &+ \underline{\lambda}^T \cdot \underline{g}(\underline{x}, \underline{u}, \underline{p}) + \underline{\mu}^T \cdot (\underline{x} - \underline{\hat{x}}) + \underline{\check{\mu}}^T \cdot (\underline{\check{x}} - \underline{x}) \\ &+ \underline{v}^T \cdot (\underline{u} - \underline{\hat{u}}) + \underline{\check{v}}^T \cdot (\underline{\check{u}} - \underline{u}) \end{aligned} \quad \dots (9.16)$$

The conditions for optimality given by the Kuhn-Tucker theorem can now be stated:

$$\underline{L}_x = 0 \quad \dots (9.17)$$

$$\underline{L}_u = 0 \quad \dots (9.18)$$

$$\underline{g}(\underline{x}, \underline{u}, \underline{p}) = 0 \quad \dots (9.19)$$

$$\underline{\mu}^T (\underline{x} - \hat{\underline{x}}) = 0 ; \quad \underline{\check{\mu}}^T (\underline{\check{x}} - \underline{x}) = 0 \quad \dots (9.20)$$

$$\underline{\check{v}}^T (\underline{u} - \hat{\underline{u}}) = 0 ; \quad \underline{\check{v}}^T (\underline{\check{u}} - \underline{u}) = 0 \quad \dots (9.21)$$

Each of the exclusion equations (9.20) and (9.21) is satisfied either because the dual variable μ_i or v_i is zero or because the corresponding constraint has been reached.

An efficient solution of the optimising equation is to impose feasible control \underline{u} not violating any constraints on \underline{x} and to associate a single dual vector

$$\underline{v} = \underline{\check{v}} - \underline{\check{v}} \quad \dots (9.22)$$

with \underline{u} . For the imposed \underline{u} , the power flow equations (9.19) yield \underline{x} .

Substitution of \underline{u} and \underline{x} into (9.17) and (9.18) provides dual vectors $\underline{\lambda}$ and \underline{v} associated with this imposed solution which may be considered optimal for the imposed \underline{u} .

For the feasible solution, artificially setting $\underline{\check{u}} = \underline{u} = \hat{\underline{u}}$ means

$$v_i = - \frac{\delta F}{\delta u_i} \quad \dots (9.23)$$

so \underline{v} measures the sensitivity of performance index F to the imposed control \underline{u} . This is then used iteratively to determine an improved control \underline{u} until constraints $\underline{\check{u}}$ or $\hat{\underline{u}}$ actually in force are reached, or until the associated dual variable becomes zero, so that no further

improvements in the control are possible.

On reaching the optimum, dual vector \underline{v} measures the sensitivity of F to relaxations $\hat{\delta u}$ or $\check{\delta u}$ in the inequality constraints on control \underline{u} . Since we have put $\underline{v} = \hat{\underline{v}} - \check{\underline{v}}$,

$$\left. \begin{aligned} v_i &= -\text{sign}(v_i) \cdot \frac{\delta F}{\delta \hat{u}_i} \\ \text{or } v_i &= -\text{sign}(v_i) \cdot \frac{\delta F}{\delta \check{u}_i} \end{aligned} \right] \dots (9.24)$$

whichever constraint is active.

In the optimal power flow, control vector \underline{u} can contain bus active and reactive generations, bus voltage levels and phase angles, together with power flows through phase shifters.

Similarly for the dual vector $\underline{\lambda}$. With fixed control \underline{u} , a small change $\delta \underline{p}$ in the parameter vector gives a small change $\delta \underline{x}$ in the state vector such that

$$\delta L = 0 = \underline{F}_x^T \cdot \underline{\delta x} + \underline{\lambda}^T \cdot [g_x] \cdot \underline{\delta x} \dots (9.25)$$

and also

$$[g_x] \cdot \underline{\delta x} + [g_p] \cdot \underline{\delta p} = 0 \dots (9.26)$$

so

$$\delta F \approx -\underline{\lambda}^T \cdot [g_x] \cdot \underline{\delta x} = \underline{\lambda}^T \cdot [g_p] \cdot \underline{\delta p} \dots (9.27)$$

since

$$\delta F \approx -\underline{F}_x^T \cdot \underline{\delta x} \dots (9.28)$$

Scalar equation (9.27) relates the change δF in performance index resulting from a change $\delta \underline{p}$ in the parameter vector.

Bus demand parameters P_{Di} and Q_{Di} only enter the i^{th} pair of load flow equations. To simplify the sensitivity relation, consider the restriction that parameter p_i enters only the i^{th} equality constraint $g_i = 0$. The matrix $[g_p]$ in this case is diagonal, equation (9.27) simplifying to

$$\delta F \approx \sum_i \lambda_i \cdot \frac{\partial g_i}{\partial p_i} \cdot \delta p_i \quad \dots (9.29)$$

in which

$$\lambda_i = \frac{\delta F}{\delta p_i} / \frac{\partial g_i}{\partial p_i} \quad \dots (9.30)$$

In the example of demand parameters P_{Di} and Q_{Di} , for which

$\frac{\partial g_i}{\partial p_i} = 1$, the dual variables are

$$\lambda_i = \frac{\delta F}{\delta P_{Di}} \quad \dots (9.31)$$

$$\lambda_i' = \frac{\delta F}{\delta Q_{Di}} \quad \dots (9.32)$$

Dual variables λ_i and λ_i' therefore measure sensitivity of the performance index to changes in real and reactive demands P_{Di} and Q_{Di} at node i . At a hydro-generation node, the generation may be considered as a negative load, or $\lambda_i = - \frac{\delta F}{\delta P_{Gi}}$.

If we impose an artificial constraint $\tilde{u} = u = \hat{u}$, any imposed control will constitute an 'optimal' solution with $\delta L = 0$. The dual vector $\underline{\lambda}$ will then measure the sensitivity of the performance index to the parameters at any feasible solution to the load flow.

9.1.3 Rescheduling the Electrothermal Subsystem

Peschon, Piercy, Tinney and Tviet [7] carried out a second order sensitivity analysis of a thermal power system and from this derived a method for schedule modification when a parameter change δp occurred. This method ignored inequality constraints and was therefore not of great practical usefulness. Dillon, Morsztyn and Tun [4] approached the problem in a different way, including inequality constraints by considering variations in the dual variables. Although involving second derivatives of the Lagrangian of the power flow equation, this formulation is essentially a first order method. The following analysis is based on the method of references [3] and [4].

Rewriting equations (9.17) and (9.18) in more convenient form

$$\underline{F}_x + \underline{\lambda}^T \cdot [g_x] + \underline{\hat{\mu}}^T - \underline{\check{\mu}}^T = 0 \quad \dots \quad (9.33)$$

$$\underline{F}_u + \underline{\lambda}^T \cdot [g_u] + \underline{\hat{v}}^T - \underline{\check{v}}^T = 0 \quad \dots \quad (9.34)$$

Expanding (9.33) and (9.34) to first order in a Taylor series and simplifying,

$$[L_{xx}] \cdot \underline{\delta x} + [L_{xu}] \cdot \underline{\delta u} + [L_{xp}] \cdot \underline{\delta p} + [g_x] \cdot \underline{\delta \lambda} + \underline{\hat{\mu}} - \underline{\check{\mu}} \approx 0 \quad \dots \quad (9.35)$$

$$[L_{ux}] \cdot \underline{\delta x} + [L_{uu}] \cdot \underline{\delta u} + [L_{up}] \cdot \underline{\delta p} + [g_u] \cdot \underline{\delta \lambda} + \underline{\hat{v}} - \underline{\check{v}} \approx 0 \quad \dots \quad (9.36)$$

Expanding equations (9.19), (9.20) and (9.21) to first order,

$$[g_x] \cdot \underline{\delta x} + [g_u] \cdot \underline{\delta u} + [g_p] \cdot \underline{\delta p} \approx 0 \quad \dots (9.37)$$

$$(\underline{x} - \hat{\underline{x}})^T \cdot \underline{\delta \hat{\mu}} + \hat{\underline{\mu}}^T \cdot \underline{\delta x} \approx 0 \quad \dots (9.38)$$

$$(\underline{\check{x}} - \underline{x})^T \cdot \underline{\delta \check{\mu}} + \check{\underline{\mu}}^T \cdot \underline{\delta x} \approx 0$$

$$(\underline{u} - \hat{\underline{u}}) \cdot \underline{\delta v} + \hat{\underline{v}}^T \cdot \underline{\delta u} \approx 0$$

$$(\underline{\check{u}} - \underline{u}) \cdot \underline{\delta v} + \check{\underline{v}}^T \cdot \underline{\delta u} \approx 0 \quad \dots (9.39)$$

So

$$\begin{bmatrix} [L_{xx}] [L_{xu}] [g_x]^T [1] [-1] [0] [0] \\ [L_{ux}] [L_{uu}] [g_u]^T [0] [0] [1] [-1] \\ [g_x] [g_u] [0] [0] [0] [0] [0] \\ [\hat{\underline{\mu}}] [0] [0] [(\underline{x} - \hat{\underline{x}})] [0] [0] [0] \\ [-\check{\underline{\mu}}] [0] [0] [0] [(\underline{\check{x}} - \underline{x})] [0] [0] \\ [0] [\hat{\underline{v}}] [0] [0] [0] [0] [(\underline{u} - \hat{\underline{u}})] [0] \\ [0] [-\check{\underline{v}}] [0] [0] [0] [0] [0] [(\underline{\check{u}} - \underline{u})] \end{bmatrix} \begin{bmatrix} \underline{\delta x} \\ \underline{\delta u} \\ \underline{\delta \lambda} \\ \underline{\delta \hat{\mu}} \\ \underline{\delta \check{\mu}} \\ \underline{\delta \hat{v}} \\ \underline{\delta \check{v}} \end{bmatrix} = - \begin{bmatrix} [L_{xp}] \\ [L_{up}] \\ [g_p] \\ [(\underline{x} - \hat{\underline{x}})_p] \\ [(\underline{\check{x}} - \underline{x})_p] \\ [(\underline{u} - \hat{\underline{u}})_p] \\ [(\underline{\check{u}} - \underline{u})_p] \end{bmatrix} \cdot \underline{\delta p} \quad \dots (9.40)$$

$$\text{where } [\hat{\underline{\mu}}] = \begin{bmatrix} \mu_1 & 0 & 0 & 0 \\ 0 & \mu_2 & 0 & 0 \\ 0 & 0 & & \\ 0 & 0 & & \mu_n \end{bmatrix} \quad \text{etc.}$$

Writing equation (9.40) as

$$[A] \cdot \underline{\delta v} = -[B] \cdot \underline{\delta p} \quad \dots (9.41)$$

the matrix $[A]$ is sparse, so the linear system of equations may be efficiently solved for $\underline{\delta v}$ by optimally ordered triangular factorisation rather than by computing the full inverse and calculating $\underline{\delta v}$ from

$$\underline{\delta v} = - [A]^{-1} \cdot [B] \cdot \underline{\delta p} \quad \dots (9.42)$$

The parameter vector \underline{p} may include line parameters such as Z_{ij} , δ_{ij} and Y_{ij} , coefficients a_{ik} of thermal plant cost functions and the active and reactive power consumptions P_{Di} and Q_{Di} at all nodes. It may also contain active generation P_{Hi} at each hydro node in the system.

Regardless of the parameters included in \underline{p} , matrix $[A]$ is unchanged, so there is little more computation required to generate additional sensitivities once the factored form of $[A]$ has been found.

To reschedule the system for altered loads, the sensitivity expression (9.40) is used to generate new values of \underline{x} , \underline{u} , $\underline{\lambda}$, $\hat{\underline{\mu}}$, $\check{\underline{\mu}}$, $\hat{\underline{v}}$, $\check{\underline{v}}$ in a series of small steps until the required change in load has been achieved. Dillon [3], for a 20% change in all active and reactive loads on a test system, generated new primal and dual variables in four equal steps of 5%, obtaining adequate accuracy for practical use.

9.1.4 Performance Index Deterioration Resulting from Inaccurate Knowledge of Parameters

Equation (9.40), the relation representing changes in the primal and dual variables in terms of a small change on the parameters, may be used in determining deterioration in the system performance index, or cost, resulting from inaccurate knowledge of the system parameters. The following derivation is based on that presented by Dillon [3].

A small change δF in system performance index $F(\underline{x}, \underline{u})$ may be represented by a Taylor series expansion.

$$\begin{aligned}\delta F = & \underline{F}_{\underline{x}} \cdot \underline{\delta x} + \underline{F}_{\underline{u}} \cdot \underline{\delta u} + \frac{1}{2} \underline{\delta x}^T \cdot [\underline{F}_{\underline{xx}}] \cdot \underline{\delta x} \\ & + \frac{1}{2} \underline{\delta u}^T \cdot [\underline{F}_{\underline{uu}}] \cdot \underline{\delta u} + \underline{\delta x}^T \cdot [\underline{F}_{\underline{xu}}] \cdot \underline{\delta u} \\ & \dots \quad (9.43)\end{aligned}$$

and expanding equation (9.10) in a Taylor series

$$\begin{aligned}& [\underline{g}_{\underline{x}}] \cdot \underline{\delta x} + [\underline{g}_{\underline{u}}] \cdot \underline{\delta u} + [\underline{g}_{\underline{p}}] \cdot \underline{\delta p} \\ & + \frac{1}{2} \underline{\delta x}^T \cdot [\underline{g}_{\underline{xx}}] \cdot \underline{\delta x} + \frac{1}{2} \underline{\delta u}^T \cdot [\underline{g}_{\underline{uu}}] \cdot \underline{\delta u} + \frac{1}{2} \underline{\delta p}^T \cdot [\underline{g}_{\underline{pp}}] \cdot \underline{\delta p} \\ & + \underline{\delta x}^T \cdot [\underline{g}_{\underline{xu}}] \cdot \underline{\delta u} + \underline{\delta u}^T \cdot [\underline{g}_{\underline{up}}] \cdot \underline{\delta p} + \underline{\delta p}^T \cdot [\underline{g}_{\underline{px}}] \cdot \underline{\delta x} \\ & = \underline{0} \quad \dots \quad (9.44)\end{aligned}$$

Now premultiplying equation (9.44) by $\underline{\lambda}^T$ and adding equations (9.38), (9.39) and (9.43) we obtain

$$\begin{aligned}\delta F = & (\underline{F}_{\underline{x}} + \underline{\lambda}^T \cdot [\underline{g}_{\underline{x}}] + \underline{\hat{\mu}}^T - \underline{\check{\mu}}^T) \cdot \underline{\delta x} + (\underline{F}_{\underline{u}} + \underline{\lambda}^T \cdot [\underline{g}_{\underline{u}}] + \underline{\hat{v}}^T - \underline{\check{v}}^T) \cdot \underline{\delta u} + \underline{\lambda}^T \cdot [\underline{g}_{\underline{p}}] \cdot \underline{\delta p} \\ & + \frac{1}{2} \underline{\delta x}^T \cdot ([\underline{F}_{\underline{xx}}] + \underline{\lambda}^T \cdot [\underline{g}_{\underline{xx}}]) \cdot \underline{\delta x} + \frac{1}{2} \underline{\delta u}^T \cdot ([\underline{F}_{\underline{uu}}] + \underline{\lambda}^T \cdot [\underline{g}_{\underline{uu}}]) \cdot \underline{\delta u} \\ & + \frac{1}{2} \underline{\delta p}^T \cdot \underline{\lambda}^T \cdot [\underline{g}_{\underline{pp}}] \cdot \underline{\delta p} + \underline{\delta x}^T \cdot ([\underline{F}_{\underline{xu}}] + \underline{\lambda}^T \cdot [\underline{g}_{\underline{xu}}]) \cdot \underline{\delta u} \\ & + \underline{\delta u}^T \cdot \underline{\lambda}^T \cdot [\underline{g}_{\underline{up}}] \cdot \underline{\delta p} + \underline{\delta p}^T \cdot \underline{\lambda}^T \cdot [\underline{g}_{\underline{px}}] \cdot \underline{\delta x} \\ & + \underline{\hat{\mu}}^T \cdot (\underline{x} - \underline{\hat{x}}) + \underline{\check{\mu}}^T \cdot (\underline{\check{x}} - \underline{x}) + \underline{\hat{v}}^T \cdot (\underline{u} - \underline{\hat{u}}) + \underline{\check{v}}^T \cdot (\underline{\check{u}} - \underline{u}) \\ & \dots \quad (9.45)\end{aligned}$$

This may be expressed in terms of first and second order partial derivatives of the Lagrangian defined by (9.16)

$$\begin{aligned}\delta F = & \underline{L}_{\underline{x}}^T \cdot \underline{\delta x} + \underline{L}_{\underline{u}}^T \cdot \underline{\delta u} + \underline{L}_{\underline{p}}^T \cdot \underline{\delta p} \\ & + \frac{1}{2} \underline{\delta x}^T \cdot [\underline{L}_{\underline{xx}}] \cdot \underline{\delta x} + \frac{1}{2} \underline{\delta u}^T \cdot [\underline{L}_{\underline{uu}}] \cdot \underline{\delta u} + \frac{1}{2} \underline{\delta p}^T \cdot [\underline{L}_{\underline{pp}}] \cdot \underline{\delta p}\end{aligned}$$

$$\begin{aligned}
& + \underline{\delta x}^T \cdot [L_{xu}] \cdot \underline{\delta u} + \underline{\delta u}^T \cdot [L_{up}] \cdot \underline{\delta p} + \underline{\delta p}^T \cdot [L_{px}] \cdot \underline{\delta x} \\
& + \underline{\delta \mu}^T \cdot (\underline{x} - \hat{x}) + \underline{\delta \mu}^T \cdot (\check{x} - \underline{x}) + \underline{\delta v}^T \cdot (\underline{u} - \hat{u}) + \underline{\delta v}^T \cdot (\check{u} - \underline{u}) \\
& \dots \quad (9.46)
\end{aligned}$$

We are determining an increase δF in performance index due to an error $\underline{\delta p}$ in measurement. In other words, the system controller receives the actual system parameters \underline{p} as $\underline{p} + \underline{\delta p}$ from which it calculates the control $\underline{u} + \underline{\delta u}$. The system, however, responds to actual parameters \underline{p} and control $\underline{u} + \underline{\delta u}$ and attains a state $\underline{x} + \underline{\delta x}$. An error $\underline{\delta p}$ in knowledge of \underline{p} therefore causes errors $\underline{\delta x}$, $\underline{\delta u}$, $\underline{\delta \lambda}$, $\underline{\delta \mu}$, $\underline{\delta \check{\mu}}$, $\underline{\delta v}$, $\underline{\delta \check{v}}$ in \underline{x} , \underline{u} , $\underline{\lambda}$, $\underline{\mu}$, $\underline{\check{\mu}}$, \underline{v} , $\underline{\check{v}}$. These errors may be calculated by solving equation (9.40) for a known error $\underline{\delta p}$.

Since the system sees actual parameters \underline{p} rather than the incorrect parameters $\underline{p} + \underline{\delta p}$ seen by the controller, the wastage δF is

$$\begin{aligned}
\delta F = & \underline{L}_x^T \cdot \underline{\delta x} + \underline{L}_u^T \cdot \underline{\delta u} + \frac{1}{2} \underline{\delta x}^T [L_{xx}] \underline{\delta x} + \frac{1}{2} \underline{\delta u}^T \cdot [L_{uu}] \cdot \underline{\delta u} \\
& + \underline{\delta x}^T \cdot [L_{xu}] \cdot \underline{\delta u} + \underline{\delta \mu}^T \cdot (\underline{x} - \hat{x}) + \underline{\delta \mu}^T \cdot (\check{x} - \underline{x}) \\
& + \underline{\delta v}^T \cdot (\underline{u} - \hat{u}) + \underline{\delta v}^T \cdot (\check{u} - \underline{u}) \quad \dots \quad (9.47)
\end{aligned}$$

This is evaluated at the optimum for which the Kuhn-Tucker necessary conditions are $\underline{L}_u = \underline{0}$, $\underline{L}_x = \underline{0}$ so

$$\begin{aligned}
\delta F = & \frac{1}{2} \underline{\delta x}^T \cdot [L_{xx}] \cdot \underline{\delta x} + \frac{1}{2} \underline{\delta u}^T \cdot [L_{uu}] \cdot \underline{\delta u} + \underline{\delta x}^T \cdot [L_{xu}] \cdot \underline{\delta u} \\
& + \underline{\delta \mu}^T \cdot (\underline{x} - \hat{x}) + \underline{\delta \mu}^T \cdot (\check{x} - \underline{x}) + \underline{\delta v}^T \cdot (\underline{u} - \hat{u}) + \underline{\delta v}^T \cdot (\check{u} - \underline{u}) \\
& \dots \quad (9.48)
\end{aligned}$$

where $\underline{\delta x}$, $\underline{\delta u}$, $\underline{\delta \mu}$, $\underline{\delta \check{\mu}}$, $\underline{\delta v}$, $\underline{\delta \check{v}}$ are given by solution of (9.40) for the parameter knowledge error $\underline{\delta p}$.

In equation (9.48), the expression

$$\underline{\delta\hat{\mu}}^T \cdot (\underline{x} - \hat{x}) + \underline{\delta\check{\mu}}^T \cdot (\check{x} - \underline{x}) + \underline{\delta\hat{v}}^T \cdot (\underline{u} - \hat{u}) + \underline{\delta\check{v}}^T \cdot (\check{u} - \underline{u}) \quad \dots \quad (9.49)$$

represents the wastage due to an inequality constraint coming into force due to the inaccurate knowledge of parameters. As an example, the term

$$\hat{\delta\mu}_i \cdot (x_i - \hat{x}_i) \quad \dots \quad (9.50)$$

represents the inequality constraint

$$x_i \leq \hat{x}_i \quad \dots \quad (9.51)$$

If constraint (9.51) is in operation at the true optimum defined by $(\underline{x}^*, \underline{u}^*, \underline{p}^*)$ then $x_i = \hat{x}_i$ and the corresponding term in (9.48) disappears. If the constraint is not in force at either the true optimum or at the actual state, then $\hat{\delta\mu}_i = 0$ and again the corresponding term in (9.48) disappears, however, if the constraint is not in force at the true optimum, but is in force at the actual state, then $(x_i - \hat{x}_i) \neq 0$ and $\hat{\delta\mu}_i \neq 0$. The resulting term, (9.50) in (9.48), represents the performance index increase due to the inequality constraint (9.51) suddenly coming into effect.

Peschon et al. [7], for a system with no inequality constraints, derived a relation for the expected value of the deterioration in performance index where the inaccuracy $\underline{\delta p}$ is a random time-uncorrelated measurement noise of zero mean. Their result may also be applied to a system with inequality constraints on \underline{x} and \underline{u} as explained below.

Equation (9.40), expressing variations in primal and dual variables in terms of error $\underline{\delta p}$ may be rewritten as a set of equations expressing each variable in terms of $\underline{\delta p}$.

$$\begin{aligned}
 \underline{\delta x} &= [\tau_1] \cdot \underline{\delta p} \\
 \underline{\delta u} &= [\tau_2] \cdot \underline{\delta p} \\
 \underline{\delta \lambda} &= [\tau_3] \cdot \underline{\delta p} \\
 \underline{\delta \hat{\mu}} &= [\tau_4] \cdot \underline{\delta p} \\
 \underline{\delta \check{\mu}} &= [\tau_5] \cdot \underline{\delta p} \\
 \underline{\delta \hat{v}} &= [\tau_6] \cdot \underline{\delta p} \\
 \underline{\delta \check{v}} &= [\tau_7] \cdot \underline{\delta p} \quad \dots (9.52)
 \end{aligned}$$

Substituting these into (9.48) we obtain

$$\begin{aligned}
 \delta F &= \frac{1}{2} \underline{\delta p}^T \cdot [\tau_1]^T \cdot [L_{xx}] \cdot [\tau_1] \cdot \underline{\delta p} + \frac{1}{2} \underline{\delta p}^T \cdot [\tau_2]^T \cdot [L_{uu}] \cdot [\tau_2] \cdot \underline{\delta p} \\
 &+ \underline{\delta p}^T \cdot [\tau_1] \cdot [L_{xu}] \cdot [\tau_2] \cdot \underline{\delta p} \\
 &+ \underline{\delta p}^T \cdot [\tau_4]^T \cdot (\underline{x} - \hat{x}) + \underline{\delta p}^T \cdot [\tau_5]^T \cdot (\check{x} - \underline{x}) \\
 &+ \underline{\delta p}^T \cdot [\tau_6] \cdot (\underline{u} - \hat{u}) + \underline{\delta p}^T \cdot [\tau_7] \cdot (\check{u} - \underline{u}) \\
 &= \underline{\delta p}^T \cdot \left\{ \frac{1}{2} [\tau_1]^T \cdot [L_{xx}] \cdot [\tau_1] + \frac{1}{2} [\tau_2]^T \cdot [L_{uu}] \cdot [\tau_2] + [\tau_1]^T \cdot [L_{xu}] \cdot [\tau_2] \right\} \cdot \underline{\delta p} \\
 &+ \underline{\delta p}^T \cdot \{ [\tau_4]^T \cdot (\underline{x} - \hat{x}) + [\tau_5]^T \cdot (\check{x} - \underline{x}) + [\tau_6]^T \cdot (\underline{u} - \hat{u}) + [\tau_7]^T \cdot (\check{u} - \underline{u}) \} \\
 &\dots (9.53)
 \end{aligned}$$

This can be expressed as

$$\delta F = \underline{\delta p}^T \cdot [K] \cdot \underline{\delta p} + \underline{\delta p}^T \cdot \underline{k} \quad \dots (9.54)$$

Since the inaccuracy $\underline{\delta p}$ is, in this case, a random time-uncorrelated measurement noise of zero mean, and defining (J) by

$$[J] \triangleq E \{ \underline{\delta p} \cdot \underline{\delta p}^T \} \quad \dots (9.55)$$

where symbol E denotes expectation, the average deterioration $E\{\delta F\}$ in performance index F caused by the random process δp is

$$\begin{aligned} E\{\delta F\} &= E\left\{\sum_i \sum_j K_{ij} \cdot \delta p_i \cdot \delta p_j + \sum_i k_i \cdot \delta p_i\right\} \\ &= \sum_i \sum_j K_{ij} \cdot J_{ij} \quad \dots (9.56) \end{aligned}$$

This equation may be written compactly as

$$E\{\delta F\} = \text{tr}[K.J] \quad \dots (9.57)$$

where the symbol "tr" denotes the trace of matrix $K.J$.

Equations (9.54) and (9.57) highlight the effects of poor measurement, estimation or forecast on performance index or cost, allowing emphasis to be placed on accuracy for the sensitive measurement in both system design and load forecasting.

9.1.5 Example Problem Illustrating Electrothermal Subsystem Sensitivity Calculations

The system used in these illustrative studies is that of Fig. 9.1, characterised by the parameters of Fig. 9.2 and bus 3 (slack bus) hydro plant performance curve of Fig. 9.3.

In each study, the appropriate parameter was perturbed and the altered results computed using the sensitivity analysis of sections 9.1.1 - 4. These altered results are compared with a full re-solution of the optimal load flow for the new value of parameter.

Studies were carried out for

- (a) 10% increase in the impedance, Z_{23} , of line 2-3
- (b) 10% increase in the loss angle, δ_{23} , of line 2-3

$$\delta F_k = \underline{a_{0k}}^T \cdot \underline{\delta u}_k + \underline{\delta u}_k^T \cdot [A_k] \cdot \underline{\delta u}_k$$

$$k = 1, 2, \dots, m \quad \dots (9.58)$$

This representation assumes unit commitment in all plants is predetermined and the electrothermal subsystem is operated optimally.

Among the hill-climbers tested in chapter five for optimisation of the hydro schedule were two second derivative methods shown to perform very well on near-quadratic hills. One of these sought to minimise the performance index, whilst the other sought to minimise the sum of squares of the gradient of the performance index with respect to the discharge rate of each hydro plant.

These methods required that a rectangular paraboloid, evaluated from a single exploratory step in the gradient direction replace the elliptic paraboloid of (9.58)

$$\delta F_k = \sum_{i=1}^{n_H-1} (A_{iik} \cdot \delta u_{ik}^2 + a_{0ik} \cdot \delta u_{ik})$$

$$k = 1, 2, \dots, m \quad \dots (9.59)$$

Since these rectangular paraboloids, one for each time interval, are available as a byproduct of the optimisation process, they immediately provide sensitivity relations between the performance index in a time interval and the plant discharge rates in that time interval.

$$\frac{dF_k}{du_{ik}} = 2A_{iik} \cdot \delta u_{ik} + a_{0ik} \quad \dots (9.60)$$

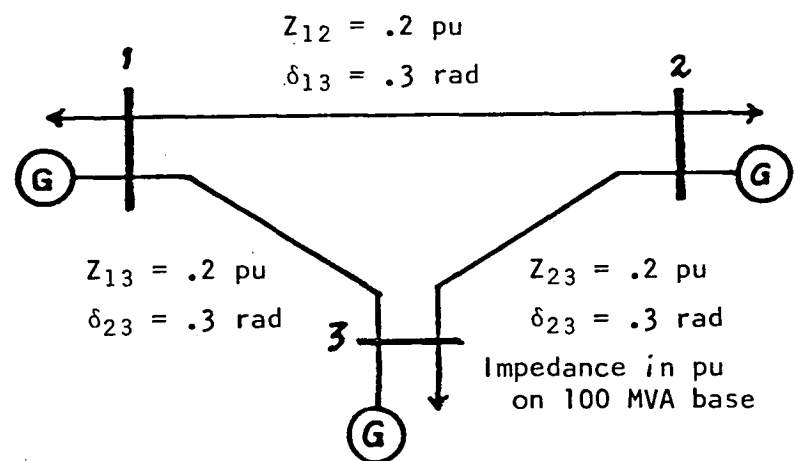


Fig. 9.1 Three node system - Node 3 is slack bus

Node	Load		Generation		Voltage	
	P_D	Q_D	P_H	Q_H	E	ϕ
1	1.25	0.18	0.02	0.2821	1.0237	-.1083
2	0.30	0.20	0.65	0.2444	1.0488	-.0239
3	0.45	0.02	1.1828	-0.0205	1.05	0

Fig. 9.2 Base case optimal load flow results

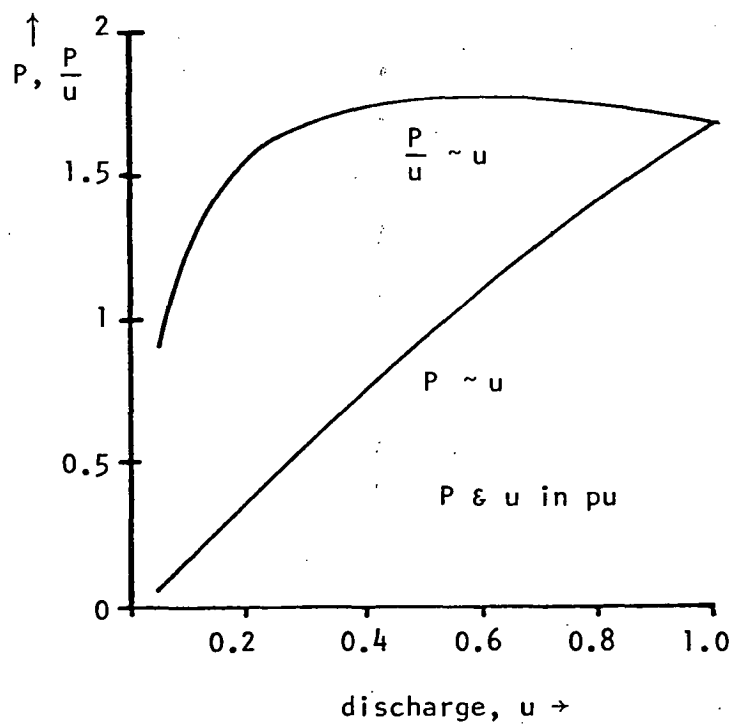


Fig. 9.3 Output and specific output curves of plant 3

Variables	P_3	Q_1	Q_2	Q_3	E_1	E_2	E_3	ϕ_1	ϕ_2
Base OLF Solution	.732779	.103039	.044418	-.041496	1.02378	1.04882	1.05	-.108286	-.023871
Dual Variables	λ_1	λ_2	λ_3	\hat{v}_1	\hat{v}_2	\hat{v}_3	F		
Base OLF Solution	1.06954	1.01493	1	0	0	.07350	.798976		

Fig. 9.4 Optimum solution of the 3-bus system of Fig. 9.1

P_i, Q_i expressed in p.u. on 100 MVA base

F_i expressed in p.u.

ϕ_i expressed in radians

Variables	P_3	Q_1	Q_2	Q_3	E_1	E_2	E_3	ϕ_1	ϕ_2
Sensitivity method	.732876	.103343	.043606	-.090669	1.02372	1.04868	1.05	-.109093	-.025450
Full optimal load flow	.732872	.103173	.043832	-.040740	1.02372	1.04871	1.05	-.109068	-.025407
Dual variables	λ_1	λ_2	λ_3	\hat{v}_1	\hat{v}_2	\hat{v}_3	F		
Sensitivity method	1.0701	1.0159	1	0	0	.070252	.799033		
Full optimal load flow	1.0701	1.0159	1	0	0	.070228	.799031		

Fig. 9.5 Comparison of sensitivity and full solution methods for a 10% increase in line impedance Z_{23}

P_i, Q_i expressed in p.u. on 100 MVA base

E_i expressed in p.u.

ϕ_i expressed in radians

Variables	P_3	Q_1	Q_2	Q_3	E_1	E_2	E_3	ϕ_1	ϕ_2
Sensitivity method	.732885	.103320	.040400	-.037734	1.02330	1.04785	1.05	-.108294	-.023708
Full optimal load flow	.732884	.103215	.040888	-.038116	1.02332	1.04791	1.05	-.108292	0.023713
Dual variables	λ_1	λ_2	λ_3	\hat{v}_1	\hat{v}_2	\hat{v}_3	F		
Sensitivity method	1.07011	1.01589	1	0	0	.070324	.799039		
Full optimal load flow	1.07011	1.01589	1	0	0	.070322	.799038		

Fig. 9.6 Comparison of sensitivity and full solution methods for a 10% increase in line loss angle δ_{23} .

P_i, Q_i expressed in p.u. on 100 MVA base

E_i expressed in p.u.

ϕ_i expressed in radians.

Variables	P_3	Q_1	Q_2	Q_3	E_1	E_2	E_3	ϕ_1	ϕ_2
Sensitivity method	.658867	.0917093	.040983	-.038854	1.02537	1.04995	1.05	-.100365	-.018425
Full optimal load flow	.659035	.092049	.040869	0.039068	1.02540	1.04995	1.05	-.100387	-.018439
Dual variables	λ_1	λ_2	λ_3	\hat{v}_1	\hat{v}_2	\hat{v}_3	F		
Sensitivity method	1.06425	1.01149	1	0	0	.061121	.755219		
Full optimal load flow	1.06428	1.01151	1	0	0	.061690	.755318		

Fig. 9.7 Comparison of sensitivity and full solution methods for a 5% increase in generation at buses 1 & 2

P_i, Q_i expressed in p.u. on 100 MVA base

E_i expressed in p.u.

ϕ_i expressed in radians

Variables	P_3	Q_1	Q_2	Q_3	E_1	E_2	E_3	ϕ_1	ϕ_2
Sensitivity method	.732079	.100933	.043227	.040495	1.03440	1.05920	1.0605	-.105983	-.023310
Full optimal load flow	.732090	.100901	.043442	-.040609	1.03439	1.05922	1.0605	0.106019	-.023323
Dual variables	λ_1	λ_2	λ_3	\hat{v}_1	\hat{v}_2	\hat{v}_3	F		
Sensitivity method	1.06800	1.01457	1	0	0	.076715	.798559		
Full optimal load flow	1.06803	1.01458	1	0	0	.068453	.798566		

Fig. 9.8 Comparison of sensitivity and full solution methods for a 1% increase in the constraint on voltage E_3

P_i, Q_i expressed in p.u. on 100 MVA base

E_i expressed in p.u.

ϕ_i expressed in radians

(c) 5% increase in hydrogeneration P_{G1} and P_{G2} at both bus 1 and bus 2

(d) 1% relaxation in all upper limits on bus voltages

These results are tabulated as Figures 9.5 - 8 and full results of the original optimal power flow are tabulated as Fig. 9.4 for comparison.

Total time for calculation of the four studies using complete optimal load flow computations was 228 seconds. Using the sensitivity method, this reduced to 29 seconds. Very close correspondence between results for the sensitivity method and the complete optimal load flow solution was obtained, verifying the theoretical results. All calculations were carried out on a small home computer programmed in basic, and differences between sensitivity and optimal load flow results are of the same order as the accumulated truncation error in the computations. The discrepancies are, in any case, much smaller than the accuracy, often worse than 1%, of measurement of system parameters and variables. The sensitivity method is therefore valid over a wider range of parameter variation than that used in these studies.

It is useful to note that the sensitivity of performance index to a change in system voltage limits was about 70 times the sensitivity to the impedance or loss angle of line 2-3.

9.2 THE HYDRO SUBSYSTEM

Chapter five and eight developed an optimisation strategy for the hydro subsystem assuming optimal operation of

the electrothermal subsystem. This optimisation strategy uses a quadratic to represent the relationship between the system performance index and hydro plant discharge rates in each time interval. These quadratics are re-evaluated at each step in the search and provide the interface between the hydro subsystem and the electrothermal subsystem both in terms of sensitivities and for the optimisation process.

As with the electrothermal subsystem, there are two major categories of sensitivity relations in the hydro subsystem, sensitivity of one hydro variable to another and sensitivity of the performance index to changes in the hydro variables, primarily total discharge and rivulet inflows. The sensitivity of the performance index to relaxation of hydro system constraints such as reservoir levels and limits on plant discharge rates are also useful, although hydro system constraints are more rigid than many of those acting on the electrothermal subsystem and may not generally be relaxed without equipment changes.

The sensitivity of one hydro variable to another will not be considered here as their form depends on the exact configuration of the hydro system and are trivial for large storage hydro plants or those which are the top plant of a cascade.

9.2.1 The Interface between Hydro and Electrothermal Subsystem Schedules

Chapter eight showed that in any time interval, the system performance index in the region of a predetermined operating point could be accurately represented by an elliptic paraboloid

$$\frac{d^2 F_k}{du_{ik}^2} = 2A_{iik} \quad \dots (9.61)$$

Off-diagonal second derivatives of performance index with respect to discharge rates are not available from this information.

Since the overall performance index is the sum of performance indices over the scheduling period, we may substitute the overall performance index f for F_k in equations (9.60) and (9.61).

9.2.2 Sensitivity of the Objective Function to Hydro Variables

The objective of the optimisation process was to minimise the sum over the scheduling period of the system performance indexes in each scheduling interval. Normally this would mean minimising the total discharge from the slack bus hydro plant of an all hydro system, or minimising the total cost of fuel and by thermal plants in a hydrothermal system. This can be expressed as

$$\text{minimise } f = \sum_{k=1}^m F_k \quad \dots (9.62)$$

subject to total discharge constraints at each hydro plant other than the slack bus plant, normally of the form

$$\underline{g}(\underline{u}, \underline{p}) = \sum_{k=1}^m \underline{u}_k - \underline{q} = \underline{0} \quad \dots (9.63)$$

for an independent plant or for the top plant of a river system, where \underline{q} is the vector of total discharge allotted by the long term schedule, or

$$g_i = \sum_{k=1}^m (u_{i,k} - (u_{i-1,k} + l_{i,k})) - q_i = 0 \quad \dots (9.64)$$

for plant i , the immediate downstream neighbour of plant $i-1$.
 in this case $l_{i,k}$ is the rivulet inflow to reservoir i in time interval k and q_i is the reservoir drawdown (or refill) allotted to reservoir i by the long term schedule.

The minimisation is also subject to a number of inequality constraints related to maximum and minimum discharge rates of each plant and to storage volumes. These may be expressed as

$$\underline{h}(\underline{u}, \underline{p}) \leq 0 \quad \dots (9.65)$$

The Kuhn-Tucker theorem provides a set of necessary conditions acting on the discharge rates \underline{u} at the minimum of f . These are normally also sufficient. A Lagrangian is defined by

$$L = f(\underline{u}, \underline{p}) + \underline{\lambda}^T \cdot \underline{g}(\underline{u}, \underline{p}) + \underline{\mu}^T \cdot \underline{h}(\underline{u}, \underline{p}) \quad \dots (9.66)$$

where $\underline{\lambda}$ and $\underline{\mu}$ are vectors of the dual variables attached to the equality and inequality constraints on the optimisation.

The conditions for optimality given by the Kuhn-Tucker theorem may now be stated

$$\underline{L}_{\underline{u}} = 0 \quad \dots (9.67)$$

$$\underline{g}(\underline{u}, \underline{p}) = 0 \quad \dots (9.68)$$

$$\underline{\mu}^T \cdot \underline{h}(\underline{u}, \underline{p}) = 0 \quad \dots (9.69)$$

The exclusion equation (9.69) is satisfied either because the dual variable μ_i is zero or because the corresponding constraint has been reached, converting the term into an equality constraint.

9.2.2.1 Interpretation of the dual variables

Multiplying equation (9.67) by \underline{u}

$$\underline{L}_u^T \cdot \underline{\delta u} = \underline{f}_u^T \cdot \underline{\delta u} + \underline{\lambda}^T \cdot [\underline{g}_u] \cdot \underline{\delta u} + \underline{\mu}^T \cdot [\underline{h}_u] \cdot \underline{\delta u} = 0 \quad \dots (9.70)$$

and expanding (9.68) and (9.69) to first order in a Taylor series,

$$[\underline{g}_u] \cdot \underline{\delta u} + [\underline{g}_p] \cdot \underline{\delta p} = 0 \quad \dots (9.71)$$

$$\underline{h}^T \cdot \underline{\delta \mu} + \underline{\mu}^T \cdot [\underline{h}_u] \cdot \underline{\delta u} + \underline{\mu}^T \cdot [\underline{h}_p] \cdot \underline{\delta p} = 0 \quad \dots (9.72)$$

considering only those constraints h_i which are active, since $\mu_i \neq 0$ for active constraints

$$[\underline{h}_u] \cdot \underline{\delta u} + [\underline{h}_p] \cdot \underline{\delta p} = 0 \quad \dots (9.73)$$

In the problem under consideration, f has been defined as a quadratic function of \underline{u} , independent of those hydro system parameters being considered here, so for small changes $\underline{\delta p}$ with corresponding control changes $\underline{\delta u}$ made to retain optimality, we can put

$$\delta f = \underline{f}_u^T \cdot \underline{\delta u} \quad \dots (9.74)$$

So, from (9.70)

$$\delta F = - \underline{\lambda}^T \cdot [g_u] \cdot \underline{\delta u} - \underline{\mu}^T \cdot [h_u] \cdot \underline{\delta u} \quad \dots \quad (9.75)$$

and substituting from (9.71) and (9.73)

$$\delta F = (\underline{\lambda}^T [g_p] + \underline{\mu}^T [h_p]) \cdot \underline{\delta p} \quad \dots \quad (9.76)$$

Considering first the equality constraints (9.63) and (9.64), the parameter p_i enters only the i^{th} constraint equation, with

$$\left. \begin{aligned} \frac{\partial g_i}{\partial p_i} &= -1 \\ \frac{\partial g_i}{\partial p_u} &= 0 \end{aligned} \right\} \quad \dots \quad (9.77)$$

and does not, in general, enter the inequality constraints. The change in performance index due to a small change in one total discharge allocation or drawdown allocation q_i is therefore

$$\delta f = \lambda_i \cdot -1 \cdot \delta q_i \quad \dots \quad (9.78)$$

$$\text{i.e.} \quad \lambda_i = - \frac{\delta f}{\delta q_i} \quad \dots \quad (9.79)$$

similarly, for variation of the independent inflow of a lower plant i in a cascade in time interval k

$$\lambda_i = - \frac{\delta f}{\delta I_{ik}} \quad \dots \quad (9.80)$$

The inequality constraints in the hydro system are of varied form, but in general, the associated dual variables measure the sensitivity of the performance index to relaxation or tightening of the active constraints. For example, consider the maximum flow restriction on plant i

$$h_i = u_i - \hat{u}_i \leq 0 \quad \dots (9.81)$$

For this constraint, the dual variable is

$$\mu_i = - \frac{\delta f}{\delta \hat{u}_i} \quad \dots (9.82)$$

These relations hold if the status of inequality constraints remains unchanged over the small change δp in the parameter vector.

9.2.2.2 Rescheduling of the hydro subsystem

More generally, expanding the Kuhn-Tucker necessary conditions of equations (9.67) - (9.69) to first order in a Taylor series.

$$[L_{uu}]. \underline{\delta u} + [L_{up}]. \underline{\delta p} + [g_u]. \underline{\delta \lambda} + [h_u]. \underline{\delta \mu} = 0 \quad \dots (9.83)$$

$$[g_u]. \underline{\delta u} + [g_p]. \underline{\delta p} = 0 \quad \dots (9.84)$$

$$\underline{h}^T. \underline{\delta \mu} + \underline{\mu}^T. [h_u]. \underline{\delta u} + \underline{\mu}^T. [h_p]. \underline{\delta p} = 0 \quad \dots (9.85)$$

so

$$\begin{bmatrix} [L_{uu}] & [g_u] & [h_u] \\ [g_u] & [0] & [0] \\ \underline{\mu}^T \cdot [h_u] & \underline{0}^T & \underline{h}^T \end{bmatrix} \cdot \begin{bmatrix} \underline{\delta u} \\ \underline{\delta \lambda} \\ \underline{\delta \mu} \end{bmatrix} = - \begin{bmatrix} [L_{up}] \\ [g_p] \\ \underline{\mu}^T \cdot [h_p] \end{bmatrix} \cdot \underline{\delta p} \quad \dots (9.86)$$

This equation,

$$[A] \cdot \underline{\delta v} = -[B] \cdot \underline{\delta p} \quad \dots (9.87)$$

in which matrix $[A]$ is sparse, may be efficiently solved for variations in the system primal and dual variables, $\underline{\delta v}$ by optimally ordered triangular factorisation.

As with the electrothermal subsystem, regardless of the parameters included in \underline{p} , matrix $[A]$ is unchanged, so little more computation is required to generated additional sensitivities once the factored increase of $[A]$ is found.

We thus have two methods available to update a schedule in the face of changed rivulet inflow predictions or total discharge allocations:

(a) Use the quadratics describing optimal operation of the electrothermal subsystem provided by the previous optimisation in a re-resolution of the hydro subsystem without updating these quadratics.

(b) Use equation (9.86) to obtain updated values of the hydro subsystem primal and dual variables.

In general method (a) is easier, as it does not involve a large matrix inversion, although if many updates from the one optimal solution are proposed, method (b) may be preferable.

9.2.3 Example Illustrating Hydro Subsystem Sensitivity Relations

For this study, the example of section 8.3 characterised by Figures 8.6 - 8.12 will be used.

Equation (9.79) shows the Lagrange multipliers used in the Newton's method search for the optimum schedule are the sensitivity of the performance index, in this case the cost of thermal power, to the total discharge of each hydro plant, i.e.

$$\lambda_i = - \frac{\delta f}{\delta q_i} \quad \dots (9.88)$$

For the example of section 8.3, at the optimum,

$$\begin{bmatrix} \frac{\delta f}{\delta q_1} \\ \frac{\delta f}{\delta q_2} \\ \frac{\delta f}{\delta q_3} \\ \frac{\delta f}{\delta q_4} \end{bmatrix} = - \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = - \begin{bmatrix} 3060 \\ 15059 \\ 3086 \\ 1406 \end{bmatrix} \quad \dots (9.89)$$

with q in $100 * (m^3/sec) * hrs$

Consider a 1% increase in total discharge for each of the four hydro plants, without any change in unit commitment. This is predicted by (9.89) to give a decrease of \$2,005 in thermal plant fuel costs over the 24 hours. The actual decrease in thermal plant fuel costs was \$1,982 calculated by complete resolution of the scheduling problem. The 0.2% discrepancy is due to the high degree of nonlinearity in the thermal plant cost characteristic.

Section 9.2.2 suggested two methods of rescheduling the power system in the face of changed rivulet inflow predictions or total discharge allocations.

The Newton's method search for the optimal schedule, using optimal load flow calculations at each step, produces quadratic approximations to optimal electrothermal subsystem operation, updating at each Newton step.

Using these quadratics in place of actual optimal load flow calculations in a Newton search, the optimal system schedule may be rapidly modified. If there is no change in the active constraints between the original optimal schedule and the updated optimal schedule, this process yields a very accurate approximation of the updated optimal schedule. For the example of section 8.3, for a change of 5% in total discharges at all hydro plants any difference between a complete solution of the optimal schedule and that using the quadratic approximations was within accumulated computation truncation error.

This exercise was repeated using the single quadratic approximation to optimal electrothermal subsystem operation which was used in the step-loading scheduling. When no constraints other than that on slack bus voltage were active, there was only a 0.01% discrepancy in thermal plant fuel cost between this method and a complete recalculation of the optimal schedule for a 5% change in total discharges at all hydro plants.

9.3 SENSITIVITY TO THE SIZE OF THE SCHEDULING INTERVALS

Initially, the time interval over which a single optimal load flow solution and a single set of hydro plant discharge rates can be applied with acceptable accuracy is related to the rate of change of system load or of individual bus loads over that interval, more frequent recalculation being needed when the system load or its distribution are changing rapidly. Since system loss characteristics are nonlinear, it is also expected that more frequent recalculation of system controls is needed at times of high system load.

Adaptive sampling in control systems has been investigated by several researchers. Three criteria need to adjust the sampling rate are those of Dorf, Farren and Phillips [5] who used the absolute value of the derivative of system error to adjust the sampling rate, of Gupta [6] who used the ratio of first and second derivatives of system error and of Bekey and Tomovic [1] who used an amplitude sensitivity approach. Bennett and Sage [2] expanded the work Bekey and Tomovic [1] to include the concept of global and local sensitivity functions with respect to the sampling interval. The global sensitivity function was used to calculate the most appropriate fixed sampling rate for the system, while the local sensitivity function was used to calculate the length of each scheduling interval on an adaptive basis.

Dillon, Morsztyn and Tun [4] applied the local sensitivity function concept to determining variable scheduling intervals for a thermal power system, obtaining a 4:1 reduction in the number

of scheduling intervals required for their test system. Two alternative criteria were formulated for determining an appropriate scheduling interval. These were to set the step length such that either

(a) A maximum per-cent change in system performance index or
(b) A maximum absolute change in system performance index is permitted in each scheduling interval. For a thermal system the performance index is defined to be the cumulative cost of fuel from the start of the scheduling period.

In their analysis, Dillon, Morsztyn and Tun ^[4] found the length of the scheduling interval to be a function only of the power generated at the start of that interval and of the allowed tolerance. While appropriate if system loads are not able to be predicted at all, this is not suited to a real system in which reasonably accurate predictions of load changes can be made.

The results for thermal power systems are not directly applicable to the variational problem posed by a hydro or hydro-thermal system. A useful result can, however, be gained if a meaningful performance index substantially independent of the variational nature of the problem is chosen. A suitable performance index is total system losses, which comprise hydro plant losses, thermal plant losses and transmission losses. If it is required that a set of scheduling intervals be generated for a scheduling period without any approximate schedule having been determined, the total system losses may be represented by a

function of predicted system or bus loads, and the criteria provided by Dillon, Morsztyn and Tun ^[4] applied to this approximation.

No numerical tests have been carried out, but it is thought that the number of scheduling intervals required to achieve a desired accuracy will be reduced by about 75%.

9.4 SUMMARY

This chapter has developed sensitivity analyses for both the electrothermal and hydro subsystems, incorporating the effects of both equality and inequality constraints acting on those systems. This analysis is important in that effective implementation of an optimal scheduling method or on-line system control requires that we determine the accuracy of modelling, measurement and prediction necessary in the power system and river systems.

Parameters with most influence on the schedule are identified by the sensitivity analysis. It also quantifies deterioration in system performance due to poor system modelling or state estimation and measurement, providing information for system planning by highlighting those system limitations which most affect system performance.

The sensitivity analysis is structured so that new schedules for not too large variations in bus loads, total discharge allocations, river flows and other parameters may be rapidly computed without the need for full optimal load flow

computations or full hydro system scheduling. This could be used to provide on-line schedule updating or control of a hydrothermal power system, taking into account both equality and inequality constraints acting on the system. The same sensitivity information used for schedule modification can be used in generating a simplified system model for medium and long term scheduling, and for updating these schedules without complete recalculation.

The variational problem of hydrothermal scheduling is solved by dividing the scheduling period into a number of scheduling intervals, then converting to a larger static problem. The number and therefore length of the scheduling intervals is of great importance in the tradeoff between accuracy and amount of computing needed for solution. The sensitivity of the performance index to the length of the scheduling intervals, together with the potential for adaptive determination of these scheduling intervals, was also discussed.

9.5 REFERENCES

- [1] Bekey, G.A. and Tomovic, R. Sensitivity of discrete systems to variations of sampling interval IEEE Trans. Auto. Control v. AC-11, pp 284-287 (1966)

- [2] Bennett, A.W. and Sage, A.P. Discrete system sensitivity and variable increment sampling, Proc. Joint Auto. Control Conf. pp 603-612 (1967)

- [3] Dillon, T.S. Problems of optimal economic operation and control of integrated (hydrothermal) and thermal power systems. Monash University, Ph.D., (1974)

- [4] Dillon, T.S., Morsztyn, K. and Tun, T. Sensitivity analysis of the problem of economic dispatch of thermal power systems.

- [5] Dorf, R.C., Farren, M.C. and Phillips, C.A. Adaptive sampling for sampled data control systems IRE Trans. on Auto. Control v. AC-7 pp 38-47 (1962)

- [6] Gupta, S.C. Adaptive gain and adaptive sampling sampled data systems, IEEE Winter General Meeting paper CP 63-120 (1963)

- [7] Peschon, J., Piercy, D.S., Tinney, W.F. and Tsvet, O.J.
Sensitivity in power systems IEEE power industry computer applications conf. (5th) pp 209-220 (1967)
- [8] Tun T. and Dillon T.S. "Sensitivity Analysis of the Problem of Economic Dispatch of Hydro-Thermal Power Systems." Proc. IFAC symposium on "Automatic Control & Protection of Electric Power Systems" Melbourne, Feb., 1977, page 237 - 241.
- [9] Dillon T.S. and Tun T. "Application of Sensitivity Methods to the Problem of Optional Control of Hydro Thermal Systems" Optimal Control Applications and Methods, Vol. 2, 117 - 143 May, 1981.

10. CONCLUSIONS AND FURTHER WORK

The subject of this thesis has been the deterministic solution of the short term scheduling problem for power systems with a significant proportion of hydrogeneration.

As detailed conclusions relating to each chapter of the thesis have been included within those chapters, only a brief summary is given here, together with overall conclusions and a discussion of proposals for extensions of this research work.

10.1 SUMMARY

An exact solution method suited to the short term scheduling of power systems with predominantly hydrogeneration has been presented. It utilises decomposition of the system into a hydro subsystem and an electrothermal subsystem. At each iteration, the schedule of the electrothermal subsystem is prepared using a set of optimal load flow solutions, incorporating system transmission losses and electrical system constraints in their most accurate form. The sensitivity information generated by the optimal load flow solutions is then used to improve the hydro subsystem schedule using either gradient or Newton search methods. A considerable amount of sensitivity information useful in very short term schedule updating is generated by both the optimal load flow solutions and the Newton's method search used for scheduling the hydro subsystem.

A feature of the proposed scheme is the use of the sub-optimal technique of step-loading, incorporating a constrained global search to preschedule the hydro subsystem, fixing hydro unit

commitment. This will facilitate convergence to a true optimal schedule. Other exact solution methods cannot assure convergence to the global optimum, but only to local extrema.

10.2 CONCLUSIONS

Historically, the solution methods for the short term scheduling of systems with a large amount of hydrogeneration have developed on different lines to those for power systems with very little, if any, hydrogeneration. The result of this separate development is that methods for hydro-based systems have lagged significantly behind those available for thermal systems because of the more complex structure of the problem and the much smaller number of researchers working on it. This thesis presents an approach based on the already highly developed solution methods available for the thermal problem and allows easy adoption of any future innovations in this area.

The usefulness of an optimal schedule from a system control operator's point of view is important. While a single optimal schedule for a hydrothermal system under closely specified conditions may be useful as a reference point, an operator needs to know the effects of deviations in loads and water inflows from those forecast, or, more rarely, the effects of machine or line outages. Also, because of the variety of considerations influencing an operator's choice of action, it is likely to be more efficient and more acceptable to provide a rapid means of choosing from a fairly small range of schedules than to incorporate all the operating considerations in one large formal optimisation. This is true also

of revision of nominal schedules on deviation from forecasts or other contingencies. Another context in which a number of scheduling options require examination is in determining load shedding strategy.

Major implications of this are that an optimal scheduling scheme should provide a wide range of cost sensitivity information and that rapid computation of feasible and near optimal schedules is of most use.

Other than the incorporation of current methods of thermal system scheduling to handle the electrothermal subsystem, the most significant feature of the scheduling scheme described in this thesis is that it ensures feasibility of all intermediate schedule obtained in the course of optimisation as well as of the final optimal schedule.

Given that an operator may well regard any new scheduling scheme with a healthy scepticism, the combination of two relatively familiar techniques, step loading and optimal load flows, each capable of providing immediately useful results in its own right, should make initial acceptance easier. The modular approach to the solution allows partial implementation initially, with the degree of implementation increasing with experience of the methods.

10.3 PROPOSALS FOR FURTHER WORK

Although the solution method presented in this thesis uses a detailed representation of both hydro and electrothermal subsystem characteristics, there are several directions in which this work can

be extended. These are outlined below.

10.3.1 Extension of the step loading method to medium term scheduling

The application of step-loading to a load duration curve as described in chapter 4 for short term unit commitment may be suited to medium and long term scheduling of a hydro or integrated power system. The basis for this is as follows:

In the long term scheduling problem, which considers one water resource cycle, usually a year, split into, say, fortnights, a predicted load curve is established. Output obtainable from run-of-river and peaking plants is subtracted according to some predetermined rule, and an allowance for transmission losses, probably based on a quadratic function of total system load, is added. The resulting load curve is then reordered to form a load vs duration curve to which the methods of chapter 4 may be applied. For the medium term scheduling problem, over say a fortnight split into individual days, storage plants and peaking plants with larger storages are scheduled by step-loading.

These long or medium term step loading schedules need not be applied directly, but may be used to generate a set of loading rules which are then applied to the predicted load or load duration curves for each scheduling subinterval.

10.3.2 The stochastic nature of river flows and system loads

River inflows, system real and reactive loads and unit

availability are stochastic in their nature. The stochastic problem is, however, so complex as to be impractical unless substantial simplifications are made in the problem formulation. Such simplifications are unacceptable in the short term scheduling solution. The stochastic representation of river inflows may, however, provide the basis for a "worst case" or "x% probable" short term solution in order to ensure feasibility of schedule if predictions are significantly in error.

Mantera [2] developed a version of his step-loading technique for short term scheduling which represented system load as a stochastic quantity. While this approach is of little use in short term scheduling, it may be adapted to the medium or long term step-loading schedule discussed in 10.3.1.

In the final chapter of his thesis, Dillon [1] noted several references related to the characterisation of the stochastic nature of loads and inflows, noting that very little work has been carried out in determining the stochastic character of loads, and that there is significant divergence of opinion in the modelling of rivers and their catchments.

It should be possible to split loads into their domestic, commercial, light industrial and heavy industrial components, then characterise each component separately, identifying the factors determining the load, and treating each major industrial load separately.

10.3.3 The state of the system

In short term scheduling, the most important factor in obtaining a truly optimal schedule is that of accurate system characteristics and load and river flow predictions. In many systems plant and transmission system characteristics are not accurately known, and the majority of system instrumentation is no better than 1% accuracy. Catchment models are also frequently accurate only for steady state conditions.

There is considerable scope for work in catchment modelling and in indirect methods of obtaining accurate system characteristics and loads in the face of inaccurate but redundant measurements. This work could, in many cases, yield larger savings than a complex optimisation based on inaccurate data.

10.3.4 General

Extensions of the work of this thesis have been proposed in application of step loading to medium and long term scheduling of a power system with significant hydrogeneration, stochastic representation of river flows and system loads to produce a worst case short term schedule to guarantee feasibility, and accurate, indirect determination of system characteristics, loads and catchment models.

There is also scope for more sophisticated treatment of storage head variations and of cascaded hydro plants in the solution method of this thesis.

10.4 REFERENCES

- [1] Dillon, T.S. Problems of optimal economic operation and control of integrated (hydrothermal) and thermal power systems. Monash University, Ph. D., 1974.

- [2] Mantera, I.G.M. Optimal load scheduling of hydro-electric power stations. University of Tasmania, Ph. D., 1972.

APPENDIX 1

DERIVATION OF THE STEP-LOADING TECHNIQUE BY PONTRYAGIN'S MAXIMUM PRINCIPLE

In this appendix, the optimal operating mode of a fixed-head hydro plant is derived using Pontryagin's maximum principle [3, 4]. It is shown that if a predetermined volume of water is allocated for discharge over the scheduling time-span, overall efficiency is maximised by step-loading the plant between two discharge rates. For an ideal system, step-loading is again found to yield an optimal schedule. Some rules must, however, be observed for effective application of this technique to non-ideal systems. This derivation was attempted by Mantera [1, 2], but as noted in Chapter 4, there was a mathematical error present, invalidating his work.

A1.1 MAXIMISATION OF THE OVERALL EFFICIENCY OF A FIXED HEAD HYDRO PLANT

Consider a hydro plant with substantially constant head over the short-term scheduling time-span $[0, T]$. The plant generation, P_G , is expressed as a nonlinear function of the discharge rate, u .

$$P_G = \phi(u) \quad \dots \quad (A1.1)$$

The optimisation objective is maximisation of the total energy generated over $[0, T]$ from a predetermined volume, Q , of water discharged. That is, maximise

$$I = \int_0^T p_G(t) \cdot dt \quad \dots \quad (A1.2)$$

subject to

$$\int_0^T u(t) \cdot dt = Q \quad \dots \quad (A1.3)$$

Q is determined from the medium- and long-term drawdown/refill policy and predicted rivulet inflows.

The optimising conditions are derived by applying Pontryagin's maximum principle [3, 4]. The differential equations characterising the system are as follows, dot-notation denoting differentiation with respect to time.

$$\begin{aligned} \dot{x}_1 &= u(t) \\ \dot{x}_2 &= p_G(t) \end{aligned} \quad \dots \quad (A1.4)$$

The following boundary conditions must be satisfied

$$\begin{aligned} x_1(0) &= x_2(0) = 0 \\ x_1(T) &= Q \end{aligned} \quad \dots \quad (A1.5)$$

and

$$I = x_2(T) \quad \dots \quad (A1.6)$$

Define a Hamiltonian

$$\begin{aligned} H &= \sum_{i=1}^2 p_i \dot{x}_i = p_1 u + p_2 \cdot \phi(u) \end{aligned} \quad \dots \quad (A1.7)$$

where adjoint variables p_i satisfy

$$\dot{p}_i = - \frac{\partial H}{\partial x_i}, \quad i = 1, 2$$

H is not an explicit function of x_i , so $\dot{p}_i = 0$. p_1 and p_2 must

therefore remain constant over $[0, T]$. The transversality conditions give

$$p_2(T) = -1 \quad \dots \quad (A1.8)$$

hence $p_2 = -1$ for all instants in $[0, T]$ so

$$H = p_1 u - \phi(u) \quad \dots \quad (A1.9)$$

The optimal $u(t)$ minimises H at all instants in $[0, T]$. If at a given instant u is at a constrained value, H is minimised by the smallest permissible u if $\frac{\partial H}{\partial u}$ is positive, or by the largest permissible u if $\frac{\partial H}{\partial u}$ is negative. At an unconstrained optimum

$$\frac{\partial H}{\partial u} = p_1 - \frac{\partial \phi}{\partial u} = 0 \quad \dots \quad (A1.10)$$

If, to satisfy the total discharge constraint (A1.3), operation at more than one discharge rate is required during $[0, T]$, then each value of u must result in the same minimal value of H .

Consider operation at one constrained discharge rate, at operating point (u_1, P_{G1}) and any number of unconstrained rates. At the first unconstrained operating point (u_2, P_{G2}) , (A1.10) gives

$$p_1 = \left. \frac{\partial \phi}{\partial u} \right|_2 \quad \dots \quad (A1.11)$$

and to maintain H , the constrained point has, from (A1.9)

$$H = \left. \frac{\partial \phi}{\partial u} \right|_2 u_1 - P_{G1} = \left. \frac{\partial \phi}{\partial u} \right|_2 u_2 - P_{G2} \quad \dots \quad (A1.12)$$

so (u_1, P_{G1}) lies on the tangent to the $P_G \sim u$ characteristic at

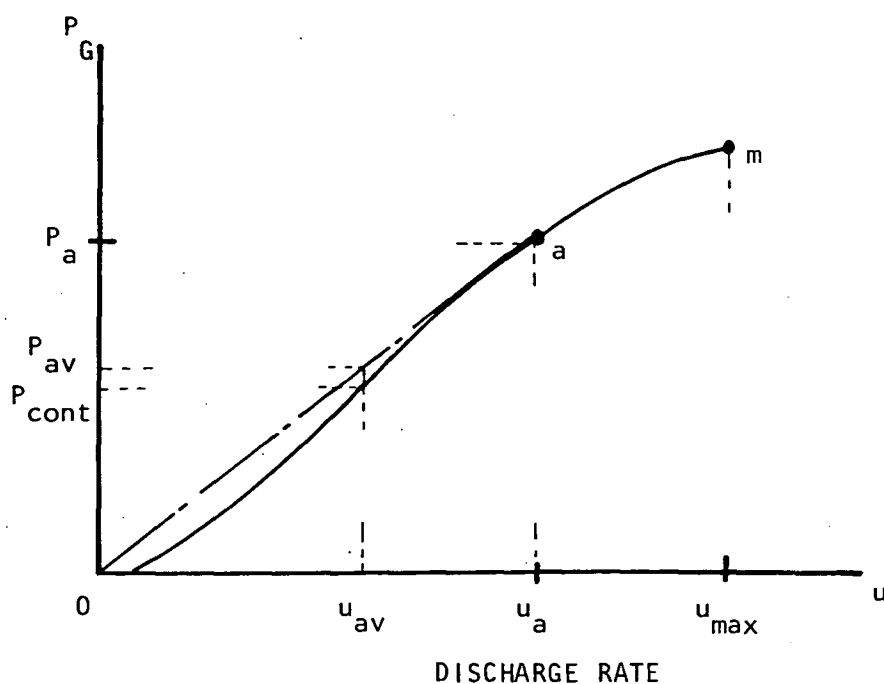


Fig A1.1 Active generation vs discharge rate characteristics of a typical single unit plant operating at fixed head (exaggerated nonlinearity)

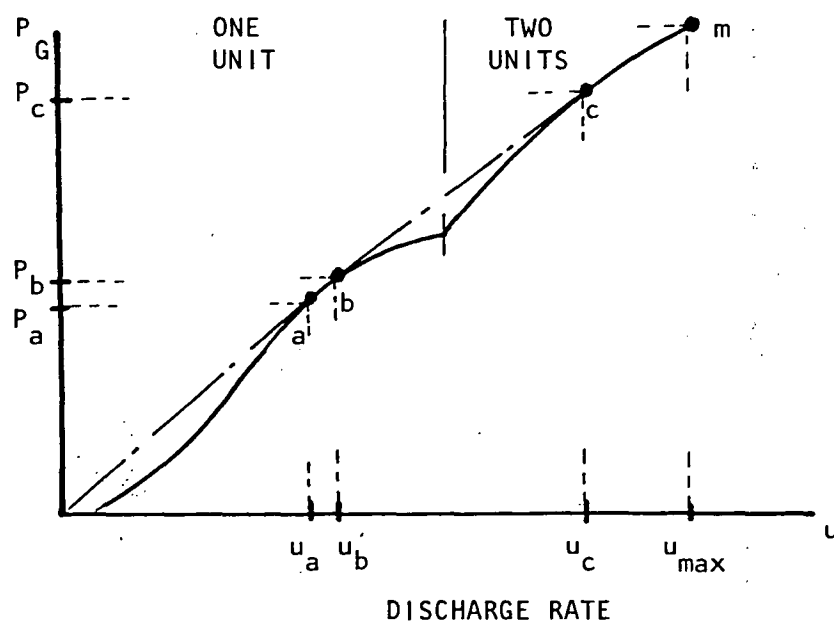


Fig A1.2 Active generation vs discharge rate characteristics of a typical two-unit plant operating at fixed head (exaggerated nonlinearity)

(u_2, P_{G_2}) . Any other unconstrained operating point must lie on the same straight line, and to satisfy (A1.10) this line is also tangent at all such points. Only one unconstrained point may therefore be used in addition to the constrained point. Both operating points are located by the requirement that $-H$, given by the intercept of the straight line on the P_G - axis should be as large as possible, and that no other point on the $P_G \sim u$ curve has lower H . That is, no other point on a line parallel to the tangent has a higher P_G - axis intercept.

By similar argument, when the discharge rate is unconstrained, the operating points must have a common tangent to the $P_G \sim u$ curve. This usually implies operation at two points close to successive efficiency peaks of a multi-unit plant. If there is no peak at a discharge rate greater than the average, then continuous operation at the average discharge rate is implied.

The resulting optimal modes of operation for one- and two-unit plants are illustrated in Fig A1.1 and are the same as those derived heuristically in Chapter 4.

In summary, to maximise the overall efficiency of a single plant subject to a preallocated volume, Q of water to be discharged within the short-term scheduling time-span $[0, T]$, the plant should be step-loaded. Continuous operation with $u = Q/T$ is regarded as a special case of step-loading. The number of switching instants plays no role in the solution, but practical considerations limit the frequency of switching.

Usually the advantages gained by keeping plant head near its maximum exceed any losses incurred in starting and shutting down

machine.

A1.2 OPTIMISATION OF THE DAILY LOAD SCHEDULE IN MULTI-PLANT SYSTEMS

Consider an all-hydro system of n plants. Q_i is the volume of water allotted for discharge by plant i within the short-term scheduling time-span $[0, T]$ as determined by the medium- and long-range storage policy. ($i = 1, 2, \dots, n-1$)

u_j is the discharge rate of plant j P_j is the generation rate of plant j J_j is the rivulet inflow to the pond of plant j P_D is the system load demand P_L is the system transmission loss	}	$j = 1, 2, \dots, n$
--	---	----------------------

The optimisation problem is formulated as:

Minimise the total depletion of the n^{th} plant storage over $[0, T]$ while satisfying the storage policy and the load balance constraint

$$P_{Gn} = P_D + P_L - \sum_{i=1}^n P_{Gi} \quad \dots \quad (A1.12)$$

Using a first-degree approximation for variation with load, the plant characteristics can be written as

$$P_{Gi} = (1 + c_i x_i) \cdot \phi_i(u_i) ; i = 1, \dots, n-1$$

$$u_n = (1 - c_n x_n) \cdot \psi_n(P_{Gn}) \quad \dots \quad (A1.13)$$

c_j is a constant, and the system dynamics are described by

$$\dot{x}_j = u_j - J_j \quad j = 1, \dots, n \quad \dots \quad (A1.14)$$

The optimisation objective is to minimise $x_n(T)$ subject to satisfying

$$\begin{aligned} x_j(0) &= 0 ; \quad j = 1, \dots, n \\ x_i(T) &= Q ; \quad i = 1, \dots, n-1 \end{aligned} \quad \dots \quad (A1.15)$$

Introduce a Hamiltonian

$$H \triangleq \sum_{j=1}^n p_j \cdot (u_j - J_j) \quad \dots \quad (A1.16)$$

$$\begin{aligned} \dot{p}_i &= - \frac{\partial H}{\partial x_i} = -p_n \cdot (1 + c_n x_n) \cdot \frac{\partial \psi_n}{\partial P_{Gn}} \cdot \frac{(-1 + \frac{\partial P_L}{\partial P_{Gi}})}{(1 - \frac{\partial P_L}{\partial P_{Gn}})} \cdot c_i \cdot \phi_i(u_i) \\ i &= 1, \dots, n-1 \quad \dots \quad (A1.17) \end{aligned}$$

$$\dot{p}_n = - \frac{\partial H}{\partial x_n} = p_n \cdot c_n \cdot \psi_n(P_{Gn})$$

The terminal boundary condition $p_n(T) = -1$ must also be satisfied.

Minimising H with respect to u_i leads to

$$\begin{aligned} p_i &= -p_n \cdot (1 + c_n x_n) \cdot \frac{\partial \psi_n}{\partial P_{Gn}} \cdot \frac{(-1 + \frac{\partial P_L}{\partial P_{Gi}})}{(1 - \frac{\partial P_L}{\partial P_{Gn}})} \cdot (1 + c_i x_i) \cdot \frac{d\phi_i}{du_i} \\ i &= 1, \dots, n-1 \quad \dots \quad (A1.18) \end{aligned}$$

Equations (A1.18) and the load constraint (A1.12) form a set of nonlinear simultaneous equations in u_i for given p_i 's. Equations (A1.17) show how the p_i 's vary with time.

The optimisation problem is then to find boundary conditions for all P_{Gj} so that $p_n(T) = -1$ and equations (A1.12) to (A1.14) are satisfied.

A1.2.1 Ideal Systems

Consider the ideal system described in section 4.1.

The plant characteristics are

$$P_{Gi} = \phi_i(u_i) ; \quad i = 1, \dots, n-1 \quad \dots \quad (A1.19)$$

$$u_n = a.P_{Gn} + b ; \quad a, b \text{ constant} \quad \dots \quad (A1.20)$$

The load demand constraint becomes

$$P_{Gn} = P_D + C - \sum_{i=1}^{n-1} P_{Gi} \quad \dots \quad (A1.21)$$

so that

$$u_n = a (P_D + C - \sum_{i=1}^{n-1} P_{Gi}) + b \quad \dots \quad (A1.22)$$

The auxiliary dependent variables are now governed by

$$\dot{p}_j = 0; \quad j = 1, \dots, n \quad \dots \quad (A1.23)$$

and the optimising conditions are

$$p_i = -a. \frac{d\phi_i}{du_i} \quad \dots \quad (A1.24)$$

The Hamiltonian can now be rewritten as

$$H = -a.C - b + a.\sum_{i=1}^{n-1} \left(u_i \cdot \frac{d\phi_i}{du_i} - P_{Gi} \right) \quad \dots \quad (A1.25)$$

The first two terms on the right of this expression are constant. Referring to section A1.1, the remaining terms are made constant by step-loading all but one of the plants. By considering the total energy drawn from storages over $[0, T]$ we see that this yields the optimal schedule. This must be so, since we have assumed the characteristic of the n^{th} plant to be linear and the total transmission loss to be independent of generation pattern.

It is necessary to select the switching instants of the first $n-1$ plants so that at no time will the n^{th} plant be required to operate beyond its capacity. If the storage policy is not to be violated, extra load arising from forecast errors must also be supplied from the n^{th} plant.

Non-ideal systems can be accommodated by the step-loading scheme as discussed in section 4.2.

A1.3 REFERENCES

- [1] Mantera, I.G.M. Optimal load Scheduling of hydroelectric power stations. Ph. D. Thesis, University of Tasmania, 1972.

- [2] Mantera, I.G.M. Optimal daily load scheduling of hydro-electric systems by means of step-loading technique. Unpublished paper 1971.

- [3] Pontryagin, L.S., Boltyanskii, V.G., Gamkrelidze, R.V., Mischenko, E.F. The mathematical theory of optimal processes. New York: Interscience publishers, 1962.

- [4] Tou, J.T. Modern control theory. New York: McGraw-Hill, 1964.

APPENDIX 2

RESULTS OF MONTE-CARLO TESTS OF HILL-CLIMB ALGORITHM PERFORMANCE

This appendix tabulates numerical results associated with tests discussed in chapter 5.

Table A2.1 Hill-Climber Performance with Perfectly Quadratic Costs

Method	Cost After Step			
	1	2	3	4
A	32.14	20.22	14.19	10.26
B	32.14	20.22	14.29	10.26
C	36.20	23.95	17.03	12.21
D	36.20	23.95	17.03	12.21
E	22.34	0.550	1.282	0.024
F	2.770	0.080	0.023	0.00027

Table A2.2 Hill-Climber Performance with Perturbed Quadratic Costs

Note: An exploratory step length of 0.1 was chosen

R	Method	Cost After Step				S.D. of Cost After Step			
		1	2	3	4	1	2	3	4
0.1	A	32.17	20.25	14.16	10.22	0.044	0.246	0.188	0.156
	B	35.03	18.93	13.48	8.776	3.393	1.962	1.896	1.743
	C	36.13	24.07	16.82	11.96	0.480	0.445	0.436	0.520
	D	38.35	22.59	13.28	8.456	3.465	5.534	4.594	3.844
	E	23.29	1.082	82.53	1.58	2.736	1.047		5.126
	F	2.281	.0859	0.0360		0.740	0.032	0.040	
0.2	A	32.27	20.26	14.08	10.08	0.208	0.513	0.422	0.345
	B	69.93	21.00	50.11	16.79	56.50	4.307	107.53	26.25
	C	36.10	24.09	16.37	11.26	0.944	0.873	1.028	1.352
	D	41.99	26.65	16.09	11.00	7.72	10.40	10.05	6.33
	E	25.40	1.14	6.66	0.424	5.77	0.685	18.49	0.942
	F	3.814	0.168	0.142	0.0059	1.585	0.113	0.426	0.018
0.4	A	32.73	20.23	13.81	9.63	1.126	1.100	1.017	1.127
	C	36.19	23.90	15.06	9.45	1.807	1.725	2.468	2.771
	D	53.97	49.84	34.08	20.42	20.11	36.86	30.65	11.89
	E	35.16	6.024	10.93	0.681	15.12	12.49	28.10	1.510
	F	9.55	0.842	.0938	.00929	5.334	0.596	0.145	.0112
0.6	A	33.81	20.20	13.58	9.831	3.717	1.711	1.586	1.446
	C	36.48	23.56	14.28	9.059	2.524	2.514	2.999	2.152
	F	27.90	6.80	1.46	0.207	21.33	6.15	2.09	0.363
0.8	A	36.32	20.29	13.44	11.47	10.74	2.25	2.41	5.82
	C	37.01	23.26	14.26	9.07	3.04	3.19	2.63	2.20
1.2	C	38.53	23.52	14.61	9.50	3.76	4.09	2.21	
2.0	C	54.01	33.42	21.08	14.61	38.12	15.39	9.85	5.34
3.0	C	55.14	42.71	32.90	29.55	20.56	19.20	16.43	22.47

Table A2.3 Hill Climber Performance on Perturbed Quadratic
Costs with Varying Exploratory Step Lengths

Perturbed Minimisation - convergence for different
exploratory step lengths.

Method	Exploratory Step Length	Cost After Step 4				S.D. After Step 4		
		R = 0	0.2	0.4	0.8	0.2	0.4	0.8
A	.01	10.26	10.17	9.54	7.05	.371	.937	1.85
	.1	10.26	10.16	9.12	8.63	.244	1.31	1.97
	.4	10.26	10.15	9.66	9.47	.244	.854	.867
	1.0	10.26	9.91	9.77	8.60	.380	1.00	1.92
	10	10.26	10.02	9.63	9.06	.565	.691	1.98
B	.01	10.25	-	-	-	-	-	-
	.1	10.26	15.16	-	-	29.8	-	-
	.4	10.26	10.25	10.22	10.05	.102	.238	.661
	1.0	10.26	9.90	9.36	8.44	.451	1.41	1.15
	10	10.26	9.80	8.11	15.12	.908	1.77	10.5
C	.01	12.21	11.70	10.11	9.88	.689	2.42	2.40
	.1	12.21	11.26	8.38	7.99	1.65	3.49	2.05
	.4	12.21	11.66	10.02	7.73	.825	2.43	2.82
	1.0	12.21	10.98	10.87	9.05	1.51	2.74	3.37
	10	12.21	11.46	10.32	9.55	1.00	2.46	2.11
D	.01	12.20	-	-	-	-	-	-
	.1	12.21	15.75	16.25	-	11.26	8.95	-
	.4	12.21	10.61	9.00	9.72	2.23	3.57	4.64
	1.0	12.21	10.65	8.37	-	2.17	2.34	-
	10	12.21	10.68	8.36	11.83	2.14	2.11	7.84
E	.01	.0239	.0433	2.76	-	.00401	7.19	-
	.1	.0240	.0400	-	-	.0818	-	-
	.4	.0242	.0751	-	-	.0877	-	-
	1.0	.0241	.0649	1.87	-	.156	6.29	-
	10	.0241	1.37	.815	-	4.43	1.77	-
F	.01	.00029	.00176	.0122	4.43	.00188	.0237	4.73
	.1	.00027	.00537	.00687	9.32	.0112	.00722	16.5
	.4	.00025	.00547	.0445	-	.0122	.128	-
	1.0	.00026	.00346	.0120	7.26	.00801	.0139	10.4
	10	.00026	.00156	.0129	2.75	.00149	.0159	4.25

Table A2.4 Hill-Climber Performance with Perturbed Quadratic Costs

Second order terms of hill halved, others adjusted.

R	Method	Cost After Step				S.D. of Cost After Step			
		1	2	3	4	1	2	3	4
0	A	36.80	25.60	19.36	15.15				
	B	32.29	21.03	14.78	10.57				
	C	36.38	25.50	17.33	12.73				
	D	36.38	25.50	17.33	12.73				
	E	0.901	.0050	.0010	$.39_{10}^{-5}$				
	F	0.896	.0049	.0010	$.65_{10}^{-5}$				
0.2	A	36.87	25.75	19.35	15.04	0.109	0.397	0.462	0.464
	B	32.30	21.01	14.70	10.47	.0156	0.156	0.195	0.200
	C	36.80	25.05	16.71	12.12	0.979	1.046	1.398	0.698
	D	36.77	25.12	14.95	9.70	1.363	1.809	3.328	3.240
	E	1.567	.0491	.0014	$.58_{10}^{-4}$	0.449	.0901	.0016	.0002
	F	1.632	.0273	$.51_{10}^{-3}$	$.46_{10}^{-5}$	0.594	.0187	$.38_{10}^{-3}$	$.36_{10}^{-4}$
0.4	A	37.03	25.69	19.12	14.27	0.263	0.762	0.778	1.032
	B	32.34	21.07	14.65	10.48	.0442	0.293	0.305	0.281
	C	37.27	25.21	16.44	10.83	1.461	1.506	2.063	2.119
	D	36.86	23.04	16.68	11.16	2.518	2.469	2.357	2.944
	E	5.442	1.002	.0414	.0037	2.866	2.348	.0575	.0057
	F	5.515	.3677	.0327	.0020	3.037	.3682	.0275	.00318
0.8	A	38.76	26.66	18.40	12.67	2.25	2.71	2.11	2.12
	B	32.47	20.91	13.92	9.181	0.168	0.528	1.144	2.010
	C	36.12	24.27	14.16	9.393	2.660	2.979	3.107	2.957
	D	40.40	41.99	13.44	7.120	5.234	34.57	4.811	5.578
1.2	A	54.84	42.35	41.38	18.42	64.24	44.54	62.16	13.99
	C	41.26	24.35	14.28	9.793	5.513	3.544	3.770	3.219

Table A2.5 Hill-Climber Performance from a Step-Loading
Starting Point

R	Method	Cost After Step				S.D. of Cost After Step			
		1	2	3	4	1	2	3	4
0	A	7.010	4.540	3.168	2.274				
	B	7.010	4.540	3.168	2.274				
	C	7.517	4.949	3.415	2.489				
	D	7.517	4.949	3.415	2.489				
	E	.0719	.0509	.0014	.0117				
	F	.0687	.2506	.0051	.6559				
0.2	A	7.036	4.550	3.138	2.227	.0341	.0373	.1017	.0993
	B	7.016	4.543	3.157	2.259	.0070	.0217	.0313	.0362
	C	7.610	4.858	3.292	2.391	.2635	.2148	.2162	.1413
	D	7.654	4.845	3.081	2.061	.3898	.3471	.4753	.4769
	E	.2246	3.5566	.0758	.0076	.0800	13.57	.2728	.0261
	F	.2132	.0109	.0013	.0001	.0768	.0092	.0015	.0001
0.4	A	7.179	4.512	2.966	1.951	.2428	.1287	.3606	.3034
	B	7.037	4.539	3.161	2.256	.0259	.0457	.0598	.0617
	C	7.682	4.868	3.111	2.058	.4537	.3437	.4134	.3241
	D	7.595	4.708	3.302	2.149	.4977	.3369	.4032	.7662
	E	.8802	.1581	.0167	.0075	.3161	.2541	.0265	.0192
	F	.8207	.0845	.0073	.0011	.3184	.0541	.0059	.0013
0.8	A	8.133	5.198	3.027	1.774	1.577	1.384	.5854	.4861
	B	7.083	4.590	3.022	2.018	.0656	.0679	.4054	.3808
	C	7.576	4.898	3.044	2.044	.6798	.3587	.3241	.4894
	D	9.038	9.726	3.374	2.137	1.445	16.14	1.892	1.067
	E	8.153	6.377	5.631	1.877	6.853	7.411	6.113	2.943
	F	6.689	4.945	3.153	.8197	6.484	7.663	4.060	1.540
1.2	A	11.60	6.597	6.427	2.671	12.24	4.515	9.333	2.222
	B	7.291	4.994	3.466	-	.3083	1.285	1.110	-
	C	8.801	5.050	3.110	2.184	1.310	.7195	.5452	.5622

A2.6

Hill Used in Monte-Carlo Tests of Hill Climbers

$$u_o = A_{oo} + \sum_{i=1}^4 u_i (A_{oi} + A_{ii} u_i + 2 \sum_{j=i+1}^4 A_{ij} u_j)$$

	A_{oo}	$\begin{bmatrix} A_{o1} \\ A_{o2} \\ A_{o3} \\ A_{o4} \end{bmatrix}$	$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ & A_{22} & A_{23} & A_{24} \\ & & A_{33} & A_{34} \\ & & & A_{44} \end{bmatrix}$
hr 1	66	$\begin{bmatrix} -6.06 \\ -7.92 \\ -6.3 \\ -1.8 \end{bmatrix}$	$\begin{bmatrix} .7 & -.05 & .01 & 0 \\ & .4 & .03 & .06 \\ & & 1 & -.1 \\ & & & .3 \end{bmatrix}$
hr 2	96.2	$\begin{bmatrix} -6.04 \\ -10.46 \\ -8.26 \\ -1.82 \end{bmatrix}$	$\begin{bmatrix} .72 & -.05 & .01 & -.01 \\ & .44 & .02 & .06 \\ & & 1 & -.08 \\ & & & .28 \end{bmatrix}$
hr 3	142.92	$\begin{bmatrix} -7.48 \\ -13.84 \\ -8.8 \\ -3 \end{bmatrix}$	$\begin{bmatrix} .75 & -.06 & .02 & 0 \\ & .5 & .02 & .05 \\ & & 1.05 & -.05 \\ & & & .25 \end{bmatrix}$
hr 4	312.72	$\begin{bmatrix} -10.3 \\ -22.92 \\ -11.48 \\ -8.12 \end{bmatrix}$	$\begin{bmatrix} .8 & -.07 & .02 & -.01 \\ & .65 & .01 & .03 \\ & & 1.08 & 0 \\ & & & .4 \end{bmatrix}$

hr 5	215.03	$\begin{bmatrix} -8.34 \\ -8.38 \\ -8.68 \\ -4.9 \end{bmatrix}$	$\begin{bmatrix} .73 & -.06 & .02 & 0 \\ & .54 & .02 & .07 \\ & & 1.04 & -.06 \\ & & & .3 \end{bmatrix}$
hr 6	145.47	$\begin{bmatrix} -7.58 \\ -13.84 \\ -8.6 \\ -3.46 \end{bmatrix}$	$\begin{bmatrix} .75 & -.06 & .02 & .01 \\ & .5 & .02 & .04 \\ & & 1.05 & -.06 \\ & & & .27 \end{bmatrix}$

A2.7 Starting Points for Hill-Climber Testing by the Monte-Carlo Method

A2.7.1 Flat Start

	hr 1	hr 2	hr 3	hr 4	hr 5	hr 6
$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$	$\begin{bmatrix} 6.166667 \\ 5.333333 \\ 4 \\ 4.5 \end{bmatrix}$	$\begin{bmatrix} 6.166667 \\ 8.333333 \\ 4 \\ 4.5 \end{bmatrix}$	$\begin{bmatrix} 6.166667 \\ 13.333333 \\ 4 \\ 4.5 \end{bmatrix}$	$\begin{bmatrix} 6.166667 \\ 25.333333 \\ 4 \\ 4.5 \end{bmatrix}$	$\begin{bmatrix} 6.166667 \\ 18.333333 \\ 4 \\ 4.5 \end{bmatrix}$	$\begin{bmatrix} 6.166667 \\ 14.333333 \\ 4 \\ 4.5 \end{bmatrix}$

A2.7.2 Step-Loading Start

	hr 1	hr 2	hr 3	hr 4	hr 5	hr 6
$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$	$\begin{bmatrix} 4.3 \\ 13 \\ 4 \\ 4 \end{bmatrix}$	$\begin{bmatrix} 4.3 \\ 13 \\ 4 \\ 4 \end{bmatrix}$	$\begin{bmatrix} 7.1 \\ 13 \\ 4 \\ 4 \end{bmatrix}$	$\begin{bmatrix} 7.1 \\ 16.5 \\ 4 \\ 7 \end{bmatrix}$	$\begin{bmatrix} 7.1 \\ 16.5 \\ 4 \\ 4 \end{bmatrix}$	$\begin{bmatrix} 7.1 \\ 13 \\ 4 \\ 4 \end{bmatrix}$

APPENDIX 3

PAPERS FOR PUBLICATION

In the course of this thesis project two papers were prepared for publication.

- [1] Jones, G.V. and Norton, J.P. Hydrothermal Optimisation by Refinement of Step-Loading Schedules. Proc. IFAC Symposium Melbourne, February, 1977.
- [2] Jones, G.V. and Norton, J.P. Feasible Short-Term Scheduling for Hydrothermal Power Systems. Unpublished.

In addition the following report has been published

Jones, G.V. On the Optimal Scheduling of Hydro-Thermal Power Systems. University of Tasmania Electrical Engineering Department Report March, 1975.

Paper 1 has been removed for copyright or proprietary reasons

FEASIBLE SHORT TERM SCHEDULING FOR HYDROTHERMAL POWER SYSTEMS

G. V. Jones

Graduate Student, Department of Electrical Engineering,
University of Tasmania, Hobart, Tasmania, Australia

J. P. Horton

Senior Lecturer, Department of Electrical Engineering,
University of Tasmania, Hobart, Tasmania, Australia

Abstract. A method is presented for the daily scheduling of hydrothermal power systems. Initially hydro plant is committed by a global search of step loading schedules. The dimensions of this search are reduced by reordering the load curve and permitting only such combinations of steps in plant loading as are found to characterise optimal step-loading schedules on the reordered curve. The transmission system and thermal generation are then coordinated and the hydro schedule refined by alternating an efficient optimal load flow computation and a hydro rescheduling algorithm using its results. The choice of algorithm, subject to stringent computing limitations which necessitate full exploitation of the behavior of the cost, is described. The use of such a scheduling technique, either off or on-line in a real system, is discussed with regard to the need for good feasible schedules even if computation must be cut short before convergence, and to the provision of sensitivity information for on-line schedule updating or for guidance of system controllers.

Keywords. Optimal control; power system control; power management; digital computer applications; electric power transmission; electric power generation; hydro plants.

INTRODUCTION

In a power system with only thermal generation, fuel stocks are usually sufficient to meet any generation schedule permitted by the instantaneous limits on generation and transmission. This is not so for hydro generation, whose energy availability is determined by water storages and stream flows and ultimately by the weather. In computing a schedule for a hydrothermal system, this extra limitation must be recognised, and it dictates the form of the computation if the proportion of hydro plant is high. Here the optimal schedule is taken to be the one supplying power demand at minimum total cost over a specified period with adequate reliability, security and quality of supply, subject to physical, operating and statutory constraints.

Optimisation of a hydrothermal system consists of a family of interacting problems characterised by differing time-scales, degrees of detail and accuracy of information. The shorter the span, the greater the detail, on the whole. The family includes future planning over a decade or so, annual optimisation over a water year, water management over a period of days or weeks, daily optimisation over a load cycle and instantaneous regulation. We shall consider the daily economic optimisation problem and its interfaces with shorter- and longer-term optimisation.

Economic hydrothermal dispatching is a variational problem because of the need to consider overall rather than just instantaneous water use, in contrast to the static or instantaneous optimisation of a purely thermal system. In the optimisation, the power system is modelled by differential-equation equality constraints describing the river dynamics, and algebraic constraints describing the transmission system. Although real and reactive loads and reservoir inflows are stochastic, in the short term they may usually be treated as deterministic without substantial penalty. Very-short-term updating of a schedule to compensate for forecasting errors may sometimes be worthwhile, and is discussed later. Otherwise the deterministic problem will be considered, with bus loads, reservoir inflows and run-of-river flows known in advance.

Historically, the daily scheduling problem was initially solved ignoring transmission losses, plant and transmission ratings and security and stability limits. Later, system losses were represented as quadratic in the active generations, using B-coefficients (Kirchmayer, 1959). More recently, optimal-control and nonlinear-programming results have been used to transform the problem into that of solving a set of differential equations constituting the conditions for optimality. As well as the difficulty of solving such equations with mixed boundary conditions in an acceptable

computing time, intermediate steps of the solution may not form feasible schedules or give a useful indication of the features of the optimal schedule. Compromise between computing effort and goodness of the schedule may therefore be difficult if not impossible. Furthermore, simultaneous solution for both thermal and hydro schedules involves a very large number of variables and inequality constraints (Dillon, 1974), causing dimensionality difficulties in systems of realistic size.

Alternatively, the solution of the integrated problem may be approached through a sequence of smaller problems in which alternately only hydro and only thermal plants are rescheduled, with reference to the same goal. Our procedure refines an initial estimated hydro schedule, employing optimal load flow (OLF) computations to schedule thermal generation, VAR allocation and regulator settings, alternating with a gradient or Newton search which uses the OLF results to improve the hydro schedule. To make the subsequent refinement effective, the initial scheduling must determine unit commitment and bring the hydro schedule into a region where the costs are locally convex functions of the discharge rates, even though the generation: discharge-rate characteristics of multi-machine hydro plants have discontinuous slopes. The commonly used technique of step-loading is suitable. The loading points are the ends of the highest common tangent to sections of the generation: discharge-rate characteristic bracketing the average power required. For given total water use, such step-loading optimises each hydro station, without reference to other stations. The schedule is optimal in the idealised case where variation of transmission losses with generation pattern, head variation and variation of thermal incremental cost with total generation are negligible. The timing of switchings between the step-loading points does not affect the average efficiency of the hydro station, but in practice it affects transmission losses and the shape of the thermal schedule, and perhaps also the thermal spinning-reserve requirements, and hence the cost.

Because of the presence of local minima (e.g. two and eight minima in tests on 3 and 4 hydro-plant systems (Jones, 1975)) a global search is necessary to find the initial step-loading schedule. Details of ways to reduce the size of the search are given, with reference to an example, in Jones and Norton (1977). Computational experience so far suggests that the scheme of alternating OLF solutions and refinements of a step-loading schedule will converge to the optimum of the integrated problem despite local minima.

By arranging the optimisation around one of the highly developed OLF algorithms, one can provide much of the sensitivity information needed to tell an operator not only the optimal schedule but also the effects of small changes on it. Equipment ratings and imposed limits, for instance on critical line flows, are also readily

incorporated.

PROBLEM FORMULATION

The short-term scheduling problem is posed as the determination of hydro and thermal generation, reactive generation from all sources and settings of on-load tap-changing (OLTC) transformers and phase-shifters, to minimise the cost of thermal generation in supplying a given load. All hydro plants are assumed to have their total reservoir depletion over the scheduling period specified in advance as part of a longer-term schedule whose calculation is not considered in detail here. Water-use constraints can be relaxed at one or more hydro plants by instead assigning a cost to water there, then treating these plants in the same way as thermal plants. All-hydro plants have often been scheduled in this way in the past (Kirchmayer, 1959). It may, however, be more convenient from the operator's viewpoint to revise the overall depletion allowed, in the event of deviation from the load forecast or variation of equipment availability. The choice of which storage depletion to revise can then be based partly on storage-management constraints and partly on immediate water value.

This value, the immediate cost of supplying power from thermal sources instead, is readily available from the Lagrange multipliers in the OLF computation, providing the contributions from any cascaded stations downstream are included. On the other hand, the future water value beyond the scheduling period is difficult to predict, as it depends on future inflows and loads and the results of storage-management policies which are influenced by such factors as maintenance and installation schedules. In practice future value may be roughly specified through a loading order for major storages, which together with a forecast of total energy demand and a regulation-storage policy for smaller storages allows daily water use for each storage to be specified tentatively. Knowledge of the short-term incremental water value permits rational modification of such a tentative depletion schedule derived from longer-term considerations.

In a purely hydro system, generation from one water-deficit storage is treated in the same way as thermal generation, its water use being the ultimate "cost" minimised. The short-term value of its water relative to that of other storages would be useful in deciding if and when to transfer to another storage for 'slack' generation.

Hydro plants are represented by their detailed power: discharge-rate characteristics including the slope discontinuities of multi-machine stations, while thermal plants are represented by their incremental cost characteristics including the effects of multiple valving and multi-machine and multi-boiler plants. The hydro characteristics embody the individual unit characteristics, pipeline, tunnel and pen-

Feasible Short Term Scheduling for Hydrothermal Power Systems

stock head-loss curves and the tailwater elevation curve.

As well as electrical constraints such as those on bus voltages, line power flows and reactive generation, hydraulic constraints are to be incorporated. They include flow constraints for run-of-river plants, delayed flow effects on cascaded plants, small-pondage constraints and downstream-flow constraints for irrigation, navigation or amenity.

The nodal equations relating bus voltage magnitudes and angles to power and reactive VA are treated in the OLF calculation as equality constraints during the optimisation. Transmission losses are modelled accurately, their dependence on nodal voltages and reactive injections not being ignored as with B-coefficient loss formulae.

SCHEDULING OF REACTIVE VA AND THERMAL GENERATION

The OLF computations, one for each hour in each step of the overall schedule refinement, determine the thermal active generation and optimise transmission conditions by adjusting all reactive generation, OLTC transformers and phase-shifters, leaving the provisional hydro schedule unaltered. They also provide dual variables (Lagrange multipliers) which are the rates of change of that hour's total generation cost with power flows across certain busbars, the coordination busbars, linking the hydro plants to the rest of the system.

The OLF solution is based on the Newton load-flow algorithm which became practicable when optimally ordered triangularisation of the Jacobian was introduced (Sato and Tinney, 1963). For a fixed system, in which transformer taps and phase-shifter settings do not change, no area interchanges within the system are specified and no VAR limits at PV nodes or voltage constraints at PQ nodes are imposed, solving the nodal equations by Newton's method typically takes four iterations from a flat start. If successive fixed-system solutions and adjustments are carried out to satisfy VAR, voltage or interchange requirements (Tinney and Hart, 1967), substantially more iterations are needed. Britton (1969) showed that interchange adjustments could be included in the Newton algorithm by appending extra interchange constraint equations to the power-flow equations. The procedure has been extended (Meyer, 1969) to include adjustment to phase-shifters and transformer taps in the Newton algorithm so as to achieve convergence in four iterations.

The algorithm generates the factorised inverse of the Jacobian of the power- and VAR-flow equations, from which can be extracted dual variables measuring the sensitivity of the cost to changes in selected control variables, usually power inflows at generator nodes, voltage magnitudes at PV and transformer-controlled busses and phase-shifter or area-interchange

power flows (Dommel and Tinney, 1968). This information is used to schedule thermal generation and optimise transmission regulation iteratively with the hydro active generation fixed.

On completion of this process, the return obtained by relaxing an active constraint on any control variable is present as a dual variable, as are the incremental costs of supplying each load, with no extra computation. These items are of obvious value to an operator contemplating an alteration. The OLF computation typically takes 10 to 20 Newton iterations, after any of which the other dual variables giving the sensitivities of the cost to power flows across the coordination busbars are available.

COORDINATION OF HYDRO AND THERMAL SCHEDULING

The incremental costs of supplying the demand can be calculated in terms of the hydro discharge rates, using the dual variables provided by the OLF solution together with the overall power: discharge-rate characteristics of the hydro plants, possibly grouping closely-connected plants whose individual reservoir depletions need not be separated.

The hydro plants are then rescheduled by a hill-climbing step with hourly flows as independent variables, constrained by the specified reservoir depletions. Each step establishes new power flows at the coordination busbars, which are held during the next OLF computation. In this way, feasible, properly constrained schedules are obtained by the end of each OLF solution. Further, each step provides a comprehensible measure of progress in the form of changes in cost and generation schedules.

The discharge-rate incremental costs give an operator a direct indication of the value of potential small changes in hydro generation at any time. The scheme therefore lends itself to intervention by an operator, who can stop the computation after any OLF solution to make his own comparison between computation time and schedule improvement. It is also possible to specify particular coordinator bus power flows in advance or in the course of the computation, on the basis of knowledge of factors affecting future flexibility of operation or security.

The choice of coordination algorithm is dictated by a need to keep the number of OLF computations small. In particular, the hill-climber must be judged on its performance over a small number of steps, the shape of the cost:discharge-rates surface must be exploited, and if second derivatives are to be used, the extra OLF computations needed must be justified by the increase in hill-climbing speed.

The hourly discharge rates u_k , $k = 1, 2, \dots, K$ determining power flows at the coordination bus-bars must give the specified total reservoir depletions

$$\sum_{k=1}^K u_k = q \quad (1)$$

Adjoining these constraints with Lagrange multipliers, the total cost f is seen to be minimised by making the hourly cost:discharge-rate gradients g_k all equal. This may be achieved by bringing 'constrained gradients' $g_k - g_j$, $k = 1, 2, \dots, K$, with j any single value from 1 to K , to zero by adjusting all hourly discharges except u_j freely, the change in u_j being determined by the constraints (1).

As f is the sum of successive hours' costs $f_k(u_k)$ which are near-quadratic, simple hill-climbers using gradients and second derivatives can be devised. Collecting the constrained gradients into a vector g_c and expanding $f(u + \delta u_c)$ up to quadratic terms in its Taylor series, the optimal constrained gradient step making the gradient at the end of the step orthogonal to the search direction is

$$\delta u_c = \hat{a} g_c^0 \cdot g_c^0 / g_c^0 \cdot g_c^0 \quad (2)$$

where g_c^0 is the gradient at the start and \hat{a} that at the end of an exploratory step of size \hat{a} in the direction of g_c^0 . The exploratory step provides implicitly the second-derivative information needed to fix the optimal step size.

Alternatively, instead of trying to bring the constrained gradients individually to zero, the hill-climber can minimise $g_c^T g_c$, the sum of the squares of the individual constrained gradients. The corresponding optimal gradient step is

$$\delta u_c = \hat{a} g_c^0 (g_c^0 - g_c^0) / (g_c^0 - g_c^0)^T (g_c^0 - g_c^0) \quad (3)$$

Other gradient algorithms deriving the second-derivative information from the change in f in the exploratory step were rejected because of ill-conditioning (Jones and Norton, 1977).

If explicit evaluation of the Hessian H_k of f with respect to each u_k is possible, Newton's method gives

$$u_k = -H_k^{-1}(b_k + \lambda) \quad (4)$$

$$\lambda = -\left[\sum_{k=1}^K H_k \right]^{-1} \left(q + \sum_{k=1}^K (H_k b_k) \right) \quad (5)$$

Here b_k is the coefficient vector of the linear terms in $f_k(u_k)$. The matrix inversions and multiplications make (4) and (5) very unattractive at first sight. However, the off-main-

diagonal terms of the Hessians are usually small. With diagonal Hessians, the equations simplify to

$$u_{ik} = -(b_{ik} + \lambda_i) / h_{iik} \quad i = 1, 2, \dots, N \quad (6)$$

and

$$\lambda_i = -(q_i + \sum_{k=1}^K b_{ik} / h_{iik}) / \sum_{k=1}^K 1 / h_{iik} \quad i = 1, 2, \dots, N \quad (7)$$

The sum of the squares of the constrained gradients may be minimised under the same assumptions, giving

$$u_{ik} = -\frac{\lambda_i}{2h_{iik}^2} - \frac{b_{ik}}{h_{iik}} \quad i = 1, 2, \dots, N \quad (8)$$

where

$$\lambda_i = -2(q_i + \sum_{k=1}^K b_{ik} / h_{iik}) / \sum_{k=1}^K 1 / h_{iik}^2 \quad i = 1, 2, \dots, N \quad (9)$$

The efficiency and robustness of these four optimisation algorithms have been tested on surfaces deviating randomly from diagonal-dominant quadratics by controlled amounts in Monte-Carlo tests (Jones and Norton, 1977). When the deviations are small, the Newton methods converge effectively in one step, as expected, but for larger deviations their performance deteriorates more rapidly than that of the gradient methods. With deviations of f and g uniformly distributed up to 40% below and above the quadratic values, the Newton algorithms still converge more rapidly, but the standard deviations of the costs using (6) and (7) are larger than those of the gradient methods. Good results are obtained from (8) and (9) for deviations up to about 60%, while the better of the gradient methods, (3), performs little worse with deviations up to 120% than on the perfect quadratic. To summarise, Newton's method is far superior for small deviations but the gradient methods are more robust; minimisation of $g_c^T g_c$ rather than f directly improves robustness of both methods and improves convergence speed for Newton's method.

CONCLUSIONS

The usefulness of an optimal-scheduling scheme from a system control operator's point of view is important. While a single optimal schedule for a hydrothermal system under closely specified conditions may be useful as a reference point, an operator needs to know in addition the effects of deviations of loads and water inflows from forecast or, more rarely, the effects of machine or line outages. Also, because of the variety of considerations influencing an operator's choice of action, it is likely to be more efficient and more acceptable to provide a quick means of choosing between a fairly small range of schedules than to attempt to incorporate all the operating considerations in one large formal optimisation. This is true also of the revision of nominal schedules on deviation from forecasts or after contingencies.

Another context in which a number of scheduling possibilities requires examination is in determining load-shedding strategy.

Two implications are that an optimal-scheduling scheme should provide a wide range of cost-sensitivity information, and that relatively quick computation of feasible and approximately optimal schedules is useful. The most significant feature of the scheme described here is its use of OLF solutions. It ensures feasibility of the schedules obtained in the course of optimisation as well as the final optimal schedule, and makes sensitivity information available with little or no extra effort. Given also that an operator might well regard any new optimal-scheduling scheme with healthy scepticism, the combination of two fairly familiar techniques, step-loading and OLF computation, each capable of giving useful results on its own, should make initial acceptance easier.

ACKNOWLEDGEMENTS

This work was supported by the Electrical Research Board. The helpfulness of discussions with Mr. G. N. Kerrison and Mr. R. L. Adams of the Hydro-Electric Commission of Tasmania is gratefully acknowledged.

REFERENCES

- BRITTON, J.P. (1969). Improved area interchange control for Newton's method load flows. *IEEE Trans. Power Appar. Sys.*, Vol. 88, No. 10, Oct. pp. 1577-1581.
- DILLON, T.S. (1974). *Problems of optimal economic operation and control of integrated hydro and thermal power systems*. Thesis (Ph.D.), Monash University.
- DOMMEL, H.W. and TINNEY, W.F. (1968). Optimal power flow solutions. *IEEE Trans. Power Appar. Sys.*, Vol. 87, No. 10, Oct. pp. 1866-1876.
- JONES, G.V. (1975). On the optimal scheduling of hydro-thermal power systems. *Electrical Engineering Dept. Report; University of Tasmania*.
- JONES, G.V. and NORTON, J.P. (1977). Hydrothermal optimisation by refinement of step-loading schedules. *IFAC Symposium on Automatic Control and Protection of Electric Power Systems*, Melbourne, I.E. Aust. Nat. Conf. Publ. No. 77/1. pp. 242-246.
- KIRCHMAYER, L.K. (1959). *Economic control of interconnected systems*. New York, Wiley.
- MEYER, W.S. (1969). *New computational techniques for the power flow and economic dispatch problems of electric utility engineering*. Thesis (Ph.D.), University of Minnesota.
- SATO, N. and TINNEY, W.F. (1963). Techniques for exploiting the sparsity of the network admittance matrix. *IEEE Trans. Power Appar. Sys.*, Vol. 82, Dec. pp. 944-950.
- TINNEY, W.F. and HART, C.E. (1967). Power flow solutions by Newton's method. *IEEE Trans. Power Appar. Sys.*, Vol. 86, No. 11, Nov. pp. 1449-1460.